MODULE 1

SOIL EXPLORATION

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1. INTRODUCTION

A fairly accurate assessment of the characteristics and engineering properties of the soils at a site is essential for proper design and successful construction of any structure at the site. The field and laboratory investigations required to obtain the necessary data for the soils for this purpose are collectively called soil exploration.

The choice of the foundation and its depth, the bearing capacity, settlement analysis & such other important aspects depend very much upon the various engineering properties of the foundation soils involved.

Soil exploration may be needed not only for the design and construction of new structures, but also for deciding upon remedial measures if a structure shows signs of distress after construction. The design and construction of highway and airport pavements will also depend upon the characteristics of the soil strata upon which they are to be aligned.

1.1 OBJECTIVES OR IMPORTANCE OF SOIL EXPLORATION

(*i*) Determination of the nature of the deposits of soil,

(*ii*) Determination of the depth and thickness of the various soil strata and their extent in horizontal direction,

(*iii*) The location of groundwater and fluctuations in Ground Water Table,

(*iv*) Obtaining soil and rock samples from the various strata,

(*v*) The determination of the engineering properties of the soil and rock strata that affect the Performance of the structure, and

(*vi*) Determination of the *in-situ* properties by performing field tests.

The different methods to know the different strata of the soil is called as methods of exploration

1.2 STAGES IN SOIL EXPLORATION

- **STAGE 1: <u>RECONNAISSANCE</u>**

This may be in the form of a field trip to the site which can reveal information on the type and behavior of adjacent sites and structures such as cracks, noticeable sags, and possibly sticking doors and windows. The type of local existing structure may influence, to a considerable extent, the exploration program and the best foundation type for the proposed adjacent structure. Since nearby existing structures must be maintained, excavations or vibrations will have to be carefully controlled. Erosion in existing cuts (or ditches) may also be observed. For highways, run off patterns, as well as soil stratification to the depth of the erosion cut, may be observed. Rocky outcrops may give an indication of the presence or the depth of bedrock.

- **STAGE 2: <u>PRELIMINARY EXPLORATION</u>**

In this phase a few borings are made or a test pit is opened to establish in a general manner the stratification, types of soil to be expected, and possibly the location of the groundwater table. One or more borings should be taken to rock, or competent strata, if the initial borings indicate the upper soil is loose or highly compressible. This amount of soil exploration is usually the extent of the site investigation for small structures. A feasibility exploration program should include enough site data and sample recovery to approximately establish the foundation design and identify the construction procedures. It is common at this stage to limit the number of good quality samples recovered and rely heavily on strength and settlement correlations using index properties such as liquid limit, plasticity index, and penetration data together with unconfined compression tests on samples recovered during penetration testing.

- **STAGE 3: <u>DETAILED EXPLORATION</u>**

Where the preliminary site investigation has established the feasibility of the project, a more detailed exploration program is undertaken. The preliminary borings and data are used as a basis for locating additional borings, which should be confirmatory in nature, and determining the additional samples required. If the soil is relatively uniform in stratification, a rather orderly spacing of borings at locations close to critical superstructure elements should be made. On occasion additional borings will be required to delineate zones of poor soil, rocky outcrops, fills, and other areas which can influence the design and construction of the foundation. Sufficient additional samples should be recovered to redefine the design and for any construction procedure required by the contractor to install the foundation. This should avoid an excessive bid for the foundation work, cost overruns, and damage to adjacent property owners from unanticipated soil conditions discovered when the excavation is opened.

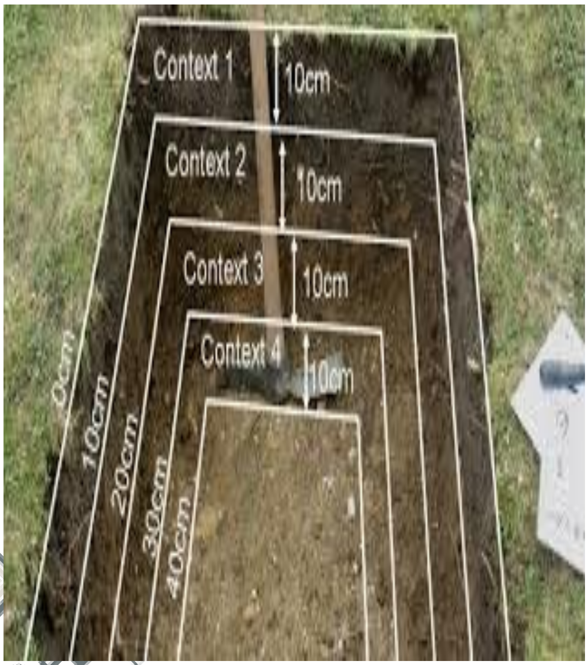
1.3 METHODS OF EXPLORATIONS

- Direct method
- Semi direct method
- Indirect method

1.3.1 DIRECT METHOD

➢ **TRIAL PITS**

Applicable to all types of soils Provide for visual examination in their natural condition. Disturbed and undisturbed soil samples can be conveniently obtained at different depths. Depth of investigation is limited to 3 to 3.5 m.



Advantages

i) Cost effective.

ii) Provide detailed information of stratigraphy.

iii) Large quantities of disturbed soils are available for testing.

iv) Large blocks of undisturbed samples can be carved out from the pits.

v) Field tests can be conducted at the bottom of the pits.

Disadvantages

i) Depth limited to about 6m.

ii) Deep pits uneconomical.

iii) Excavation below groundwater and into rock difficult and costly.

iv) Too many pits may scar site and require backfill soils.

Limitations

i) Undisturbed sampling is difficult

ii) Collapse in granular soils or below ground water table

1.3.2 SEMI DIRECT METHOD:

➢ **BORING TECHNIQUES**

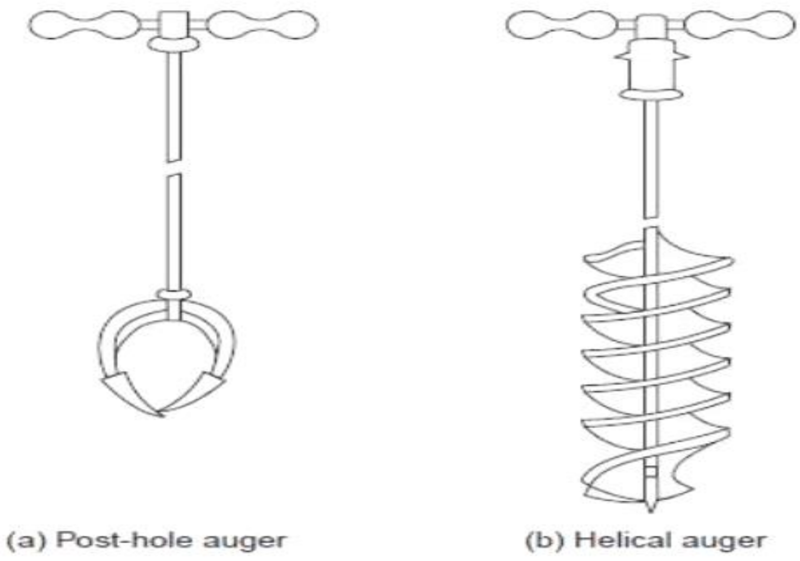
Making or drilling bore holes into the ground with a view to obtaining soil or rock samples from specified or known depths is called 'boring'.

The common methods of advancing bore holes are:

1. Auger boring
2. Auger and shell boring
3. Wash boring
4. Percussion drilling
5. Rotary drilling

❖ **AUGER BORING**

'Soil auger' is a device that is useful for advancing a bore hole into the ground. Augers may be hand-operated or power-driven; the former are used for relatively small depths (less than 7 m), while the latter are used for greater depths. The soil auger is advanced by rotating it while pressing it into the soil at the same time. It is used primarily in soils in which the bore hole can be kept dry and unsupported. As soon as the auger gets filled with soil, it is taken out and the soil sample collected. Two common types of augers, the post hole auger and the helical auger.

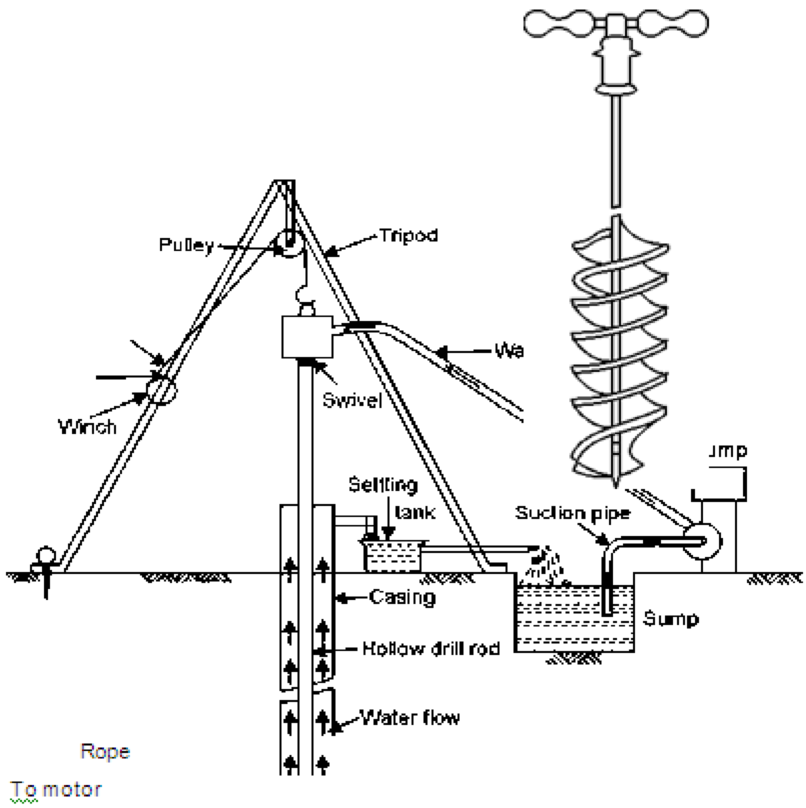


❖ **AUGER AND SHELL BORING**

If the sides of the hole cannot remain unsupported (filled soils), then the soil presented besides should be prevented from sliding in by means of a pipe known as 'shell' or 'casing'. The casing is to be driven first and then the auger; whenever the casing is to be extended, the auger has to be withdrawn, this being an impediment to quick progress of the work. An equipment called a 'boring rig' is employed for power-driven augers, which may be used up to 50 m depth (A hand rig may be sufficient for borings up to 25 m in depth). Casings may be used for sands or stiff clays. Soft rock or gravel can be broken by chisel bits attached to drill rods. Sand pumps are used in the case of sandy soils.

❖ **WASH BORING**

Wash boring is commonly used for exploration below ground water table for which the auger method is unsuitable. This method may be used in all kinds of soils except those mixed with gravel and boulders. The set-up for wash boring is shown in Fig.



Initially, the hole is advanced for a short depth by using an auger. A casing pipe is pushed in and driven with a drop weight. The driving may be with the aid of power. A hollow drill bit is screwed to a hollow drill rod connected to a rope passing over a pulley and supported by a tripod. Water jet under pressure is forced through the rod and the bit into the hole.

This loosens the soil at the lower end and forces the soil-water suspension upwards along the annular surface between the rod and the side of the hole. This suspension is led to a settling tank where the soil particles settle while the water overflows into a sump. The water collected in the sump is used for circulation again.

The soil particles collected represent a very disturbed sample and is not very useful for the evaluation of the engineering properties. Wash borings are primarily used for advancing bore holes; whenever a soil sample is required, the chopping bit is to be replaced by a sampler.

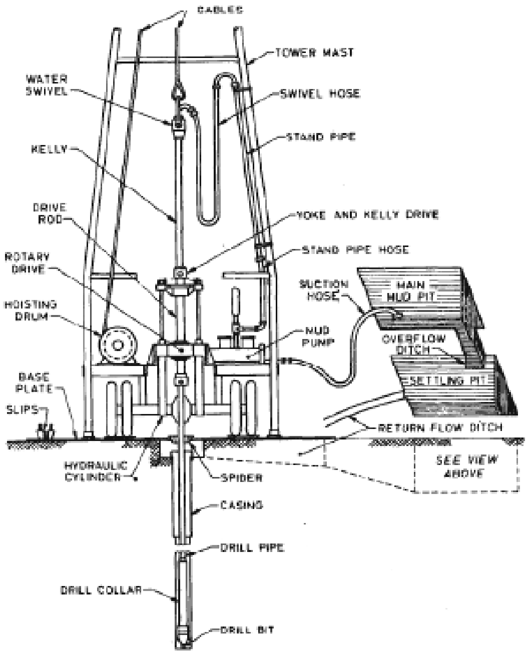
The change of the rate of progress and change of color of wash water indicate changes in soil strata.

❖ **PERCUSSION DRILLING**

A heavy drill bit called 'churn bit' is suspended from a drill rod or a cable and is driven by repeated blows. Water is added to facilitate the breaking of stiff soil or rock. The slurry of the pulverized material is bailed out at intervals. The method cannot be used in loose sand and is slow in plastic clay. The formation gets badly disturbed by impact.

❖ **ROTARY DRILLING**

This method is fast in rock formations. A drill bit, fixed to the lower end of a drill rod, is rotated by power while being kept in firm contact with the hole. Drilling fluid or bentonite slurry is forced under pressure through the drill rod and it comes up bringing the cuttings to the surface. Even rock cores may be obtained by using suitable diamond drill bits. This method is not used in porous deposits as the consumption of drilling fluid would be prohibitively high.

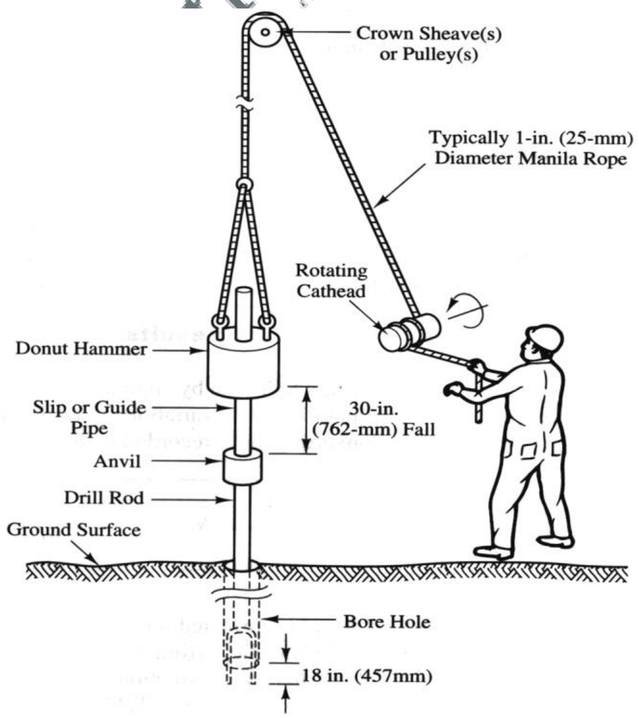


1.3.3 INDIRECT METHODS

1. Sounding or penetration tests
2. Geophysical methods

- Seismic method
- Electrical resistivity method.

1.3.3.1 PENETRATION TEST



SPT "IS: 2131-1986 — standard penetration test".

- Generally used for cohesion-less soils.
- To determine relative density, angle of shearing resistance, UCC.
- A bore hole is made using drilling tools and a hammer of weight 63.5kg falling from the height of 750 mm at the rate of 30 blows/minute.
- After reaching the specified depth, the drilling tool is replaced by a split spoon sampler to collect soil sample.
- First 150 mm penetration is taken as seating drive and the no. of blows required for that penetration is discarded
- No of blows required for next 300mm penetration after seating drive is taken as standard penetration number (N)
- No of blows greater than 50 are taken as refusal and the test is discontinued
- Corrections are applied to the observed N value

CORRECTION TO N VALUE

1. Dilatancy Correction
2. Overburden correction

Of these, overburden correction is applied first and to that corrected value, dilatancy Correction is applied

DILATANCY CORRECTION:

- Due to the presence of fine sand and silt below the water table, negative pore pressure develops which increases, the observed N value.
- If N'<15 or N=15 , N' = N,
- N = 15 + [0.5(N' – 15)]

OVER BURDEN CORRECTION:

- Soils having the same relative density will show higher
- N value at greater depth due to presence of over burden.
- Cohesion less soils are greatly affected by confining pressure. Hence N value is corrected $\sigma <= 280$ kN/m^2.
- N= N' – [350/(σ+70)]

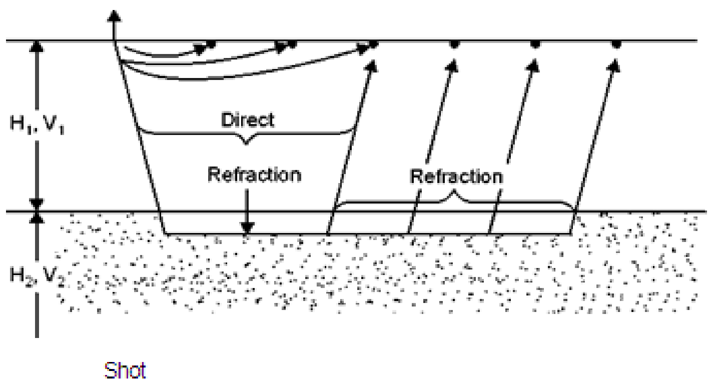
1.3.3.2 GEOPHYSICAL METHOD

- **SEISMIC REFRACTION METHOD**

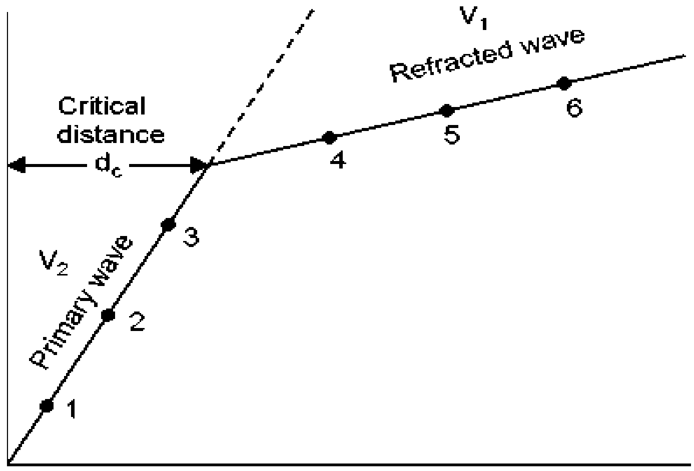
When a shock or impact is made at a point on or in the earth, the resulting seismic (shock or sound) waves travel through the surrounding soil at speeds related to their elastic characteristics.

A shock may be created with a sledge hammer hitting a strike plate placed on the ground or by detonating a small explosive charge at or below the ground surface. The radiating shock waves are picked up by detectors, called ‘geophones’, placed in a line at increasing distances, d1, d2, ..., from the origin of the shock (The geophone is actually a transducer, an electromechanical device that detects vibrations and converts them into measurable electric signals). The time required for the elastic wave to reach each geophone is automatically recorded by a ‘seismograph’.

Some of the waves, known as direct or primary waves, travel directly from the source along the ground surface or through the upper stratum and are picked up first by the geophone. If the sub soil consists of two or more distinct layers, some of the primary waves travel down-wards to the lower layer and get refracted as the surface. If the underlying layer is denser, the refracted waves travel much faster. As the distance from the source and the geophone increases, the refracted waves reach the geophone earlier than the direct waves. Figure 18.15 shows the diagrammatic representation of the travel of the primary and the refracted waves. The distance of the point at which the primary and refracted waves reach the geophone simultaneously is called the ‘critical distance’ which is a function of the depth and the velocity ratio of the strata.



The results are plotted as a distance of travel versus time graph, known as the 'time-travel graph'. A simple interpretation is possible if each stratum is of uniform thickness and each successively deeper stratum has a higher velocity of transmission.

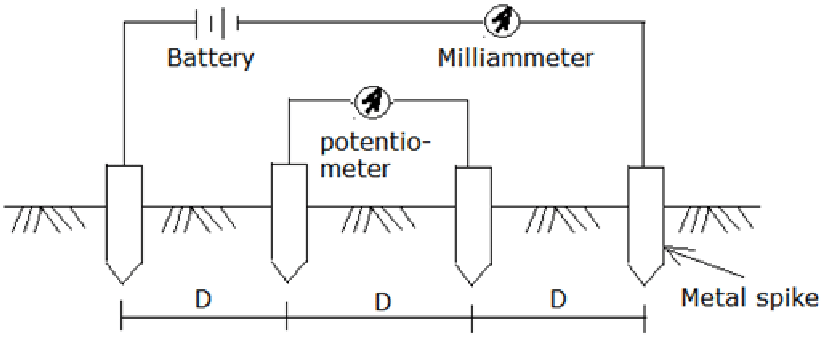


The reciprocal of the slope of the travel-time graph gives the velocity of the wave. The travel-time graph in the range beyond the critical distance is flatter than that in the range within that distance. The velocity in this range also can be computed in a similar manner. The break in the curve represents the point of simultaneous arrival of primary and refracted waves, or the critical distance. The travel-time graph appears somewhat as shown in fig.

- **ELECTRICAL RESISTIVITY METHOD**

Electrical resistivity method is based on the difference in the electrical conductivity or the electrical resistivity of different soils. Resistivity is defined as resistance in ohms between the opposite phases of a unit cube of a material.

This method is based on the measurement and recording of changes in the mean resistivity or apparent specific resistance of various soils. The test is done by driving four metal spikes to act as electrodes into the ground along a straight line at equal distances. This is shown in the figure.



Direct voltage is applied between the two outer potentiometer electrodes and then mean for the potential drop between the inner electrodes is calculated.

Mean resistivity (ohm-cm):

$$\rho = 2\pi D \frac{E}{I} = 2\pi D R$$

Where D = distance between the electrodes (cm)

E= potential drop between outer electrodes (volts)

I= current flowing between outer electrodes (amperes)

R= resistance (ohms)

Resistivity mapping: This method is used to find out the horizontal changes in the sub soil, the electrodes kept at a constant spacing, are moved as a group along the line of tests.

Resistivity sounding: This method is used to study the vertical changes; the electrode system is expanded, about a fixed central point by increasing the spacing gradually from an initial small value to a distance roughly equal to the depth of exploration desired.

1.4 TYPES OF SAMPLES

Broadly speaking, samples of soil taken out of natural deposits for testing may be classified depending upon the degree of disturbance caused during sampling operations as:

- Disturbed samples
- Undisturbed samples

'Undisturbed', in this context, is a purely relative term, since a truly undisturbed sample can perhaps be never obtained as some little degree of disturbance is absolutely inevitable even in the best method of sampling devised till date.

A **disturbed** sample is that in which the natural structure of the soil gets modified partly or fully during sampling, while an undisturbed sample is that in which the natural structure and other physical properties remain preserved.

Disturbed samples may be further subdivided as:

(i) Non-representative samples, and

(ii) Representative samples.

Non-representative samples consist of mixture of materials from various soil or rock strata or are samples from which some mineral constituents have been lost or got mixed up.

Soil samples obtained from auger borings and wash borings are non-representative samples. These are suitable only for providing qualitative information such as major changes in subsurface strata.

Representative samples contain all the mineral constituents of the soil, but the structure of the soil may be significantly disturbed. The water content may also have changed. They are suitable for identification and for the determination of certain physical properties such as Atterberg limits and grain specific gravity.

1.5 SAMPLING TECHNIQUES

'Soil Sampling' is the process of obtaining samples of soil from the desired depth at the desired location in a natural soil deposit, with a view to assessing the engineering properties of the soil for ensuring a proper design of the foundation. The ultimate aim of the exploration methods described earlier, it must be remembered, is to obtain soil samples besides obtaining all relevant information regarding the strata. The devices used for the purpose of sampling are known as 'soil samplers'.

Soil samples are classified as 'thick wall' samplers and 'thin wall' samplers. Split spoon sampler (or split tube sampler) is of the thick-wall type, and 'Shelby' tubes are of the thin-wall type.

Depending upon the mode of operation, samplers may be classified as the open drive sampler, stationary piston sampler and rotary sampler.

Open drive sampler can be of the thick wall type as well as of the thin wall type. The head of the sampler is provided with valves to permit water and air to escape during driving. The check valve helps to retain the sample when the sampler is lifted. The tube may be seamless or may be split in two parts; in the latter case it is known as the split tube or split spoon sampler.

Stationary piston sampler consists of a sampler with a piston attached to a long piston rod extending up to the ground surface through drill rods. The lower end of the sampler is kept closed with the piston while the sampler is lowered through the bore hole. When the desired elevation is reached, the piston rod is clamped, thereby keeping the piston stationary, and the sampler tube is advanced further into the soil. The sampler is then lifted and the piston rod clamped in position. The piston prevents the entry of water and soil into the tube when it is being lowered, and also helps to retain the sample during the process of lifting the tube. The sampler is, therefore, very much suited for sampling in soft soils and saturated sands.

Rotary samplers are of the core barrel type (USBR, 1960) with an outer tube provided with cutting teeth and a removable thin liner inside. It is used for sampling in stiff cohesive soils.

- **Split-Spoon Sampler**

The split spoon sampler is basically a thick-walled steel tube, split length wise. The sampler as per BIS (IS: 2131-1986—Standard Penetration Test for soils) is shown in Fig.

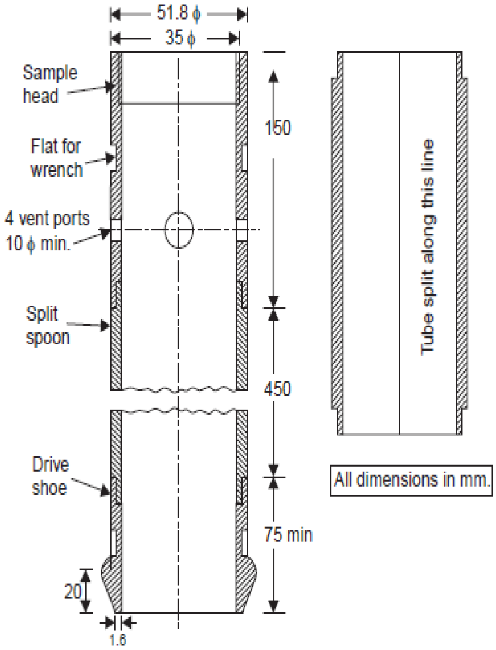


Fig: Split spoon sampler (I.S.)

A drive shoe attached to the lower end serves as the cutting edge. A sample head may be screwed at the upper end of split spoon. The standard size of the spoon sampler is of 35 mm internal and 50.8 mm external diameter. The sampler is lowered to the bottom of the bore hole by attaching it to the drill rod. The sampler is then driven by forcing it into the soil by blows from a hammer. The assembly of the sampler is then extracted from the hole and the cutting edge and coupling at the top are unscrewed. The two halves of the barrel are separated and the sample is thus exposed. The sample may be placed in a glass jar and sealed, after visual examination. If samples need not be examined in the field, a liner is inserted inside the split spoon. After separating the two halves, the liner with the sample is sealed with wax.

➢ **Thin-Walled Samplers**

Thin-walled sampler, as per BIS (I.S.: 2132-1986 Code of Practice) for Thin walled Tube Sampling of Soils), is shown in Fig.

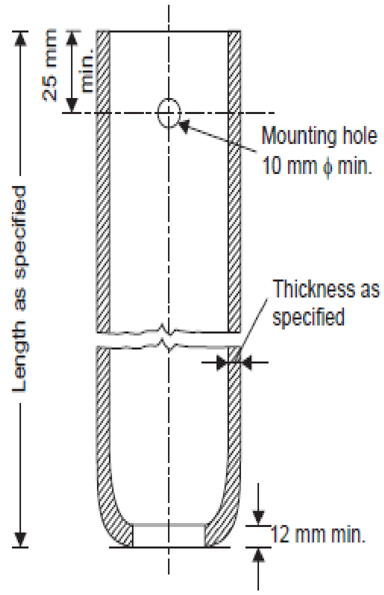


Fig: Thin-walled sampler (I.S.)

The sampling tube shall be made of steel, brass, or aluminum. The lower end is leveled to form a cutting edge and is tapered to reduce wall friction. The salient dimensions of three of the sampling tubes are given in following table:

Inside diameter, mm	38	70	100	Area ratio in this case is
Outside diameter, mm	40	74	106	$\left(\dfrac{D_e^2 - D_i^2}{D_i^2}\right)$
Minimum effective length available for soil sample, mm	300	450	450	where D_e = External dia.
Area Ratio, A_r%	10.9	11.8	12.4	D_i = Internal dia.

Note: Sampling tubes of intermediate or larger diameters may also be used.

After having extracted the sample in the same manner as in the case of split spoon type, the tube is sealed with wax on both ends and transported to the laboratory.

1.6 SAMPLE DISTURBANCE

The design features of a sampler, governing the degree of disturbance of a soil sample are the dimensions of the cutting edge and those of the sampling tube, the characteristics of the non return valve and the wall friction. In addition, the method of sampling also affects the sample disturbance. The lower end of a sampler with the cutting edge is shown in Fig.

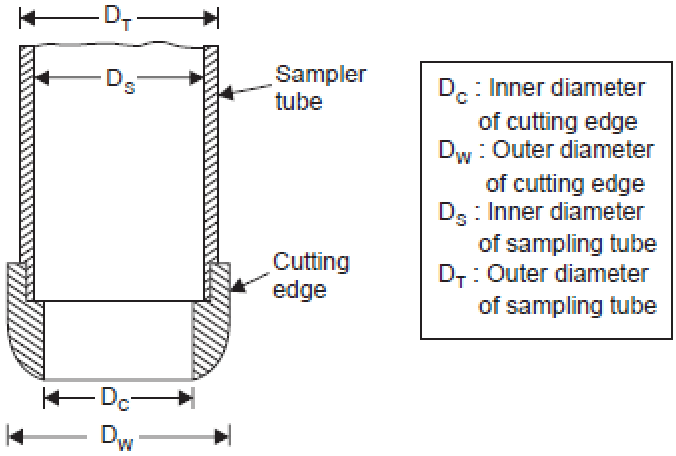


Fig: Sampling tube with cutting edge

Area Ratio, $$A_r = \frac{(D_w^2 - D_c^2)}{D_c^2} \times 100\%$$

Inside clearance, $$C_I = \frac{(D_s - D_c)}{D_c} \times 100\%$$

Outside clearance, $$C_o = \frac{(D_w - D_T)}{D_T} \times 100\%$$

The walls of the sampler should be kept smooth and properly oiled to reduce wall friction in order that sample disturbance be minimized. The non-return valve should have a large orifice to allow the air and water to escape quickly and easily when driving the sampler. Area ratio is the most critical factor which affects sample disturbance; it indicates the ratio of displaced volume of soil to that of the soil sample collected. If A_r is less than 10%, the sample disturbance is supposed to be small. *Ar* may be as high as 30% for a thick wall sampler like split spoon and may be as low as 6 to 9% for thin wall samplers like Shelby tubes. The inside clearance, *CI*, should not be more than 1 to 3%, the outside clearance *Co* should also not be much greater than *CI*. Inside clearance allows for elastic expansion of the soil as it enters the tube, reduces frictional drag on the sample from the wall of the tube, and helps to retain the core. Outside clearance facilitates the withdrawal of the sample from the ground.

The recovery ratio L_r = Recovered length of sample/ Penetration length of sampler

$L_r = 1$ indicates good recovery

$L_r < 1$ indicates soil is compressed

$L_r > 1$ indicates soil is swelled

1.7 STABILIZATION OF BORE HOLE

For geotechnical engineering purposes the borehole is not drilled to its maximum depth in a single operation. The drilling operation is to be stopped at regular intervals for in-situ testing and sampling. At all time, the boreholes once drilled must remain as a borehole i.e. the soils on the sites of the borehole must not cave in and fill up the borehole. Maintaining the integrity of the borehole is known as stabilization of borehole.

The following methods are commonly employed in practice to stabilize the borehole:

1. Self supportive.
2. Stabilizing by filling with water.
3. Stabilizing by filling with drilling mud.
4. Stabilizing by casing.

SELF SUPPORTIVE

Borehole in clay are usually self supportive. Above the water table such soil has high apparent cohesion and below the water table enough undrained shear strength to prevent the soil caving in the borehole.

Silty soil above the water table are also self supportive because of apparent cohesion due to negative pore water pressure. Below the water table, negative pore water pressure gets eliminated and borehole needs suitable support.

STABILIZING BY FILLING WITH WATER

When the GWT is at a higher elevation than that of water in the borehole, water flows into the borehole and seepage forces tends to push the soil into the borehole.

Seepage forces can be used to keep the soil particles in their original position if the direction of flow is reversed. This can be achieved by filling the boreholes with water to a level above that of GWT. Boreholes in sites and sandy silts can be stabilized by this method.

STABILIZING BY FILLING WITH DRILLING MUD

Drilling mud is water with bentonite clay. The stabilizing capacity of a drilling mud lies in the fact that it provides a coating of bentonite on the walls of borehole. This coating of high plastic material helps coarse grained particle to stick with each other and prevents falling into the borehole. Since, the level of drilling mud in the borehole is kept higher than GWT, no flow occurs into the borehole. The disadvantage of using drilling mud is that it is messy.

STABILIZING BY CASING

Casing pipe method of stabilizing borehole is adopted in medium and coarse sand, soft clays and whenever the other methods do not work. The hole is drilled for a short distance, the drilling rod is withdrawn and the casing pipe having an outside diameter equal to the diameter of borehole is pushed into the borehole. Drilling the borehole and penetrating the casing pipe is to be continued upto the desired depth. The water level in the pipe is to be maintained at a level higher than GWT.

1.8 BORING LOG

Information on subsurface conditions obtained from the boring operation is typically presented in the form of a boring record, commonly known as "boring log". A continuous record of the various strata identified at various depths of the boring is presented. Description or classification of the various soil and rock types encountered, and data regarding ground water level have to be necessarily given in a pictorial manner on the log. A "field" log will consist of this minimum information, while a "lab" log might include test data presented alongside the boring sample actually tested.

Sometimes a subsurface profile indicating the conditions and strata in all borings in series is made. This provides valuable information regarding the nature of variation or degree of uniformity of strata at the site. This helps in delineating between "good" and "poor" area.

The standard practice of interpolating between borings to determine conditions surely involves some degree of uncertainty. A site plan showing the disposition of the borings should be attached to the records.

S.No.	*Type of foundation*	*Depth of exploration*
1.	Isolated spread footings or raft or adjacent footings with clear spacing equal or greater than four times the width	One and half times the width
2.	Adjacent footings with clear spacing less than twice the width	One and half times the length
3.	Adjacent rows of footings	
	(*i*) With clear spacing between rows less than twice the width	Four and half times the width
	(*ii*) With clear spacing between rows greater than twice the width	Three times the width
	(*iii*) With clear spacing between rows greater than or equal to four times the width	One and half times the width
4.	Pile and Well foundations	One and half times the width of structure from bearing level (toe of pile or bottom of well)
5.	Road cuts	Equal to the bottom width of the cut
6.	Fill	Two metres below the ground level or equal to the height of the fill whichever is greater

RECORD OF BORING [IS : 1892-1979]

Name of boring organization:

Bored for ....
Ground level..............................
Type of boring..............................
Diameter of boring..............................
Inclination: Vertical..............................
Bring: ....

Location-site ....
Boring No. ....
Soil sampler used ....
Date started ....
Date completed ....
Recorded ....

Description of strata	*Soil classification*	*Thickness of stratum*	*Depth from GL*	*R.L. of lower contact*	*Samples*			*GWL*	*Re-marks*
					Type	*No.*	*Depth and thickness of sample*		
Fine to medium sand with practically no binder	SP		1 m 2 m 2.7 m		Undisturbed	1	1 m 1.4 m 1.7 m 2 m		
Silty clays of medium plasticity no coarse or medium sands	CI		3 m 4 m 5 m		Undisturbed	2	3 m 4 m 4.3 m 5 m	Not struck upto 6 m depth	

BOREHOLE SPACING- GUIDELINES

The following table gives the general guidelines for the spacing of boreholes:

Type of project	Spacing in(m)	Depth
Multi storied Building	10-30	10m
Industrial plant	20-60	6 m if single story
Residential Buildings	250-500	6-10m
Dams and dikes	40-80	20m

1.9 ESTIMATION OF DEPTH OF GWT (HVORSLEV'S METHOD)

As per the Hvorslev's method, water table level can also be located in a borehole used for soil investigation. That type of bore hole should have a casing to stabilize the sides. It uses almost the same technique; the rise in water level determines the water level locations. However, there is a slight difference. Unlike Casagrande piezometer method, this method consists of hailing the water out of the casing and observing the rate of rise of the water level in the casing at different intervals of time until the rise in Water level becomes negligible.

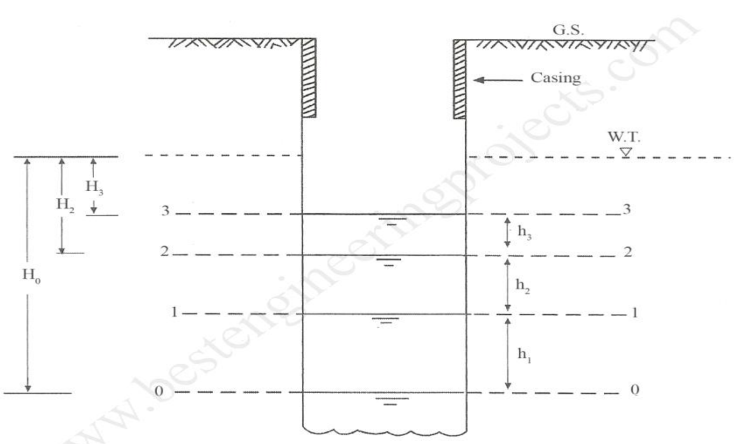


Fig 1 Water Level Location by Rising Water Table Method

Figure shows the rise of water level in the borehole at different time intervals. The height of water above the levels 0 – 0, 2 – 2 and 3 – 3 is calculated from the following equations.

H_o = Height above level 0 – 0 = $\frac{h_1^2}{h_1 - h_2}$

H_2 = Height above level 2 – 2 = $\frac{h_2^2}{h_1 - h_2}$

H_3 = Height above level 3 – 3 = $\frac{h_3^2}{h_2 - h_3}$

Let the corresponding depth of water level below the ground surface be h_{w1}, h_{w2}, h_{w3}

$h_{w1} = H_W - H_O$

$h_{w2} = H_W - (h1+h2+H_2)$

$h_{w3} = H_W - (h1+h2+h3+H_3)$

Here H_W is the depth of water level in casing from the ground surface at the beginning of the test.

Normally, $h_{w1} = h_{w2} = h_{w3}$…...

If not, take the average of the same.

1.10 <u>DRAINAGE AND DEWATERING</u>

Dewatering involves controlling groundwater by pumping, to locally lower groundwater levels in the vicinity of the excavation. The simplest form of dewatering is sump pumping, where groundwater is allowed to enter the excavation where it is then collected in a sump and pumped away by robust solids handling pumps. Sump pumping can be effective in many circumstances, but seepage into the excavation can create the risk of instability and other construction problems. To prevent significant groundwater seepage into the excavation and to ensure stability of excavation side slopes and base it may be necessary to lower groundwater levels in advance of excavation.

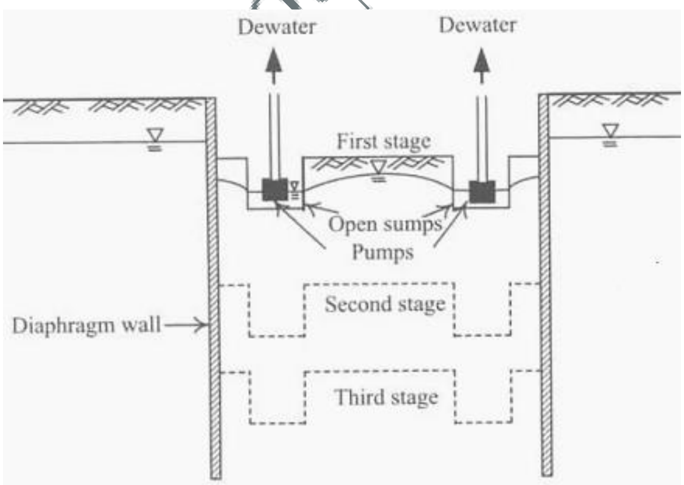
OBJECTIVES OF DEWATERING OR DRAINAGE

- To keep working place dry like excavation for dams, building foundations and tunnels.
- To stabilize natural or constructed slopes
- To treat granular soils by reducing their compressibility
- To decrease lateral pressures on retaining walls or foundation
- To improve bearing capacity of foundation soils
- To reduce liquefaction potential due to seismic activity
- To prevent migration of soil particles by groundwater (phenomenon of piping)
- To reduce surface erosion

METHODS OF DEWATERING

1. DRAINAGE AND SUMPS

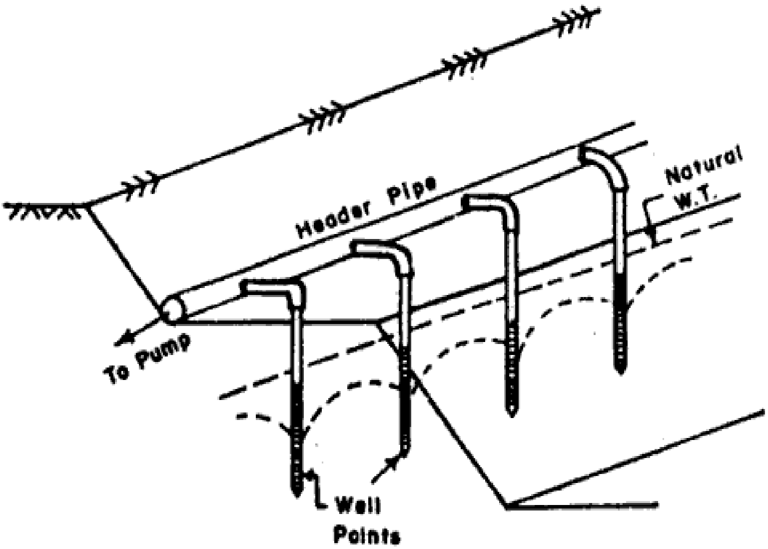
This is the most common and economical method of dewatering as gravity is the main playing force. Sump is created in the excavated area into which the surrounding water converges and accumulates facilitating easy discharge of water through robust solid handling pumps.



Its application is however confined to the areas where soil is either gravelly or sandy. Since the bottom of the sump is situated at a level lower than that of the excavation bottom, it will abridge the seepage way along which groundwater from outside seeps into the excavation zone and as a result the exit gradient of the sump bottom will be larger than that on the excavation surface. If the excavation area is large, several sumps may be placed along the longer side or simply use a long narrow sump which is called a ditch.

2. **WELLPOINT METHOD**

- A series of wells of required depth are created in the vicinity of the excavated area from where the water has to be pumped out. The wells are arranged either in a line or a rectangular form where the well points are created at a distance of at least 2m from each other.
- Riser pipes or dewatering pipes are then installed into those closely spaced wells which on the surface are connected to a flexible swing pipe which is ultimately appended to a common header pipe that is responsible for discharging the water away from the site. The purpose of using a flexible swing pipe is just to provide a clear view of what is being pumped and the purpose of header pipe is to create suction as well as discharge the water off the working area.



- One end of the header pipe is connected to a vacuum pump which draws water through notches in the well point. The water then travels from the well points through the flexible swing pipe into the header pipe to the pump. It is then discharged away from the site or to other processes to remove unwanted properties such as contaminants.

- The drawdown using this method is restricted to around five to six meters below the well point pump level. If a deeper drawdown is required, multiple stages of well points must be used.

3. MULTISATGE WELL POINT SYSTEM

If the water table must be lowered more than 5 or 6 m but the permeability is relatively low, so that the quantity of water per well is too small for economical use of large-diameter deep-well pumps, a jet-eductor well-point system may be advantageous. The jet-eductor pump, located immediately above the well point, is operated by water furnished to the eductor under high pressure. The well point is established at the bottom of a casing at least 100 mm in diameter, in which are installed the pressure and discharge pipes for the eductor. The casing may be surrounded by a filter.

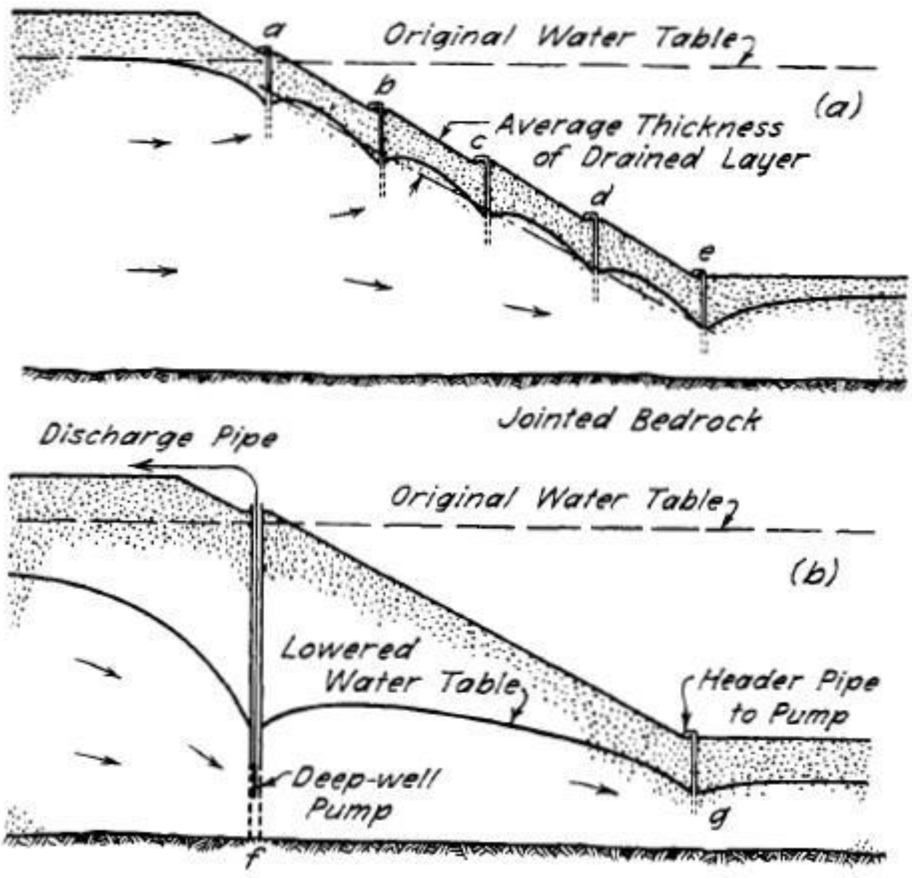


Fig: Well point system

4. **VACUUM METHOD**

If the average effective grain size D_{10} of the soil is smaller than about 0.05 mm, the methods of gravity drainage described in the preceding paragraphs fail to produce the desired results, because the water is retained in the voids of the soil by capillary forces. However, the stabilization of very fine-grained soils can be accomplished at least gradually by maintaining a vacuum in the filters that surround the well points (shown in Fig.).

Before the vacuum is produced, both the upper surface of the fine-grained layer and the soil surrounding the filter are acted on by the pressure *u*, of the atmosphere, approximately 100 kPa. After the vacuum has been produced, the pressure on the soil around the filters is almost equal to zero, the high degree of cohesion that the soil acquired during pumping. When the vacuum method is used, the well points are commonly spaced at 1 m.

The pumping equipment is the same as that for draining soils of medium permeability. One 150-mm pump is used for every 150 m of the length of a row of well points. In addition, one or two vacuum pumps are attached to the header-pipe lines. One 15 kW motor is sufficient to operate the entire pump aggregate. Because of the low permeability of the soil, the water pump discharges for short periods only. The vacuum pumps operate continually. The success of the method depends to a large extent on the quality of the vacuum pumps and on the skill and experience of the foreman whereas that on the surface of the layer remains equal to *u*, Consequently, water is gradually squeezed out of the soil into the evacuated filters until the effective pressure in the soil adjoining the row of well points has increased by an amount equal to the atmospheric pressure. At the same time the shearing resistance of the soil increases by an amount equal to *u*, tan +', where 9' is the angle of internal friction of the soil. This process is very similar to the stiffening of clay due to desiccation. The following method is used to construct a filter that can be evacuated. After the well point is jetted into the ground, the pressure of the jetting water is increased until a hole with a diameter of 250 to 300 mm has been scoured out. While the water is still flowing, sand is shoveled into the hole until the top of the sand reaches an elevation a meter or so below the surface of the fine-grained stratum. The water is then turned off, and the rest of the hole is filled with clay or silt which acts as a seal. The results that can be obtained by this method are illustrated by Fig. which shows an open excavation in organic silt with an average effective grain size less than 0.01 mm. Ninety-five % of the soil passed the No. 200 screen (0.07 mm). The bottom of the excavation was about *5* m below the original water table. Before pumping, the silt was so soft that the crane, visible in the background, had to be moved on a runway of heavy timbers. After two weeks of pumping the soil was so stiff that the sides of the excavation did not require lateral support. The distinct marks left by the excavating tools indicate an increase in strength and generally by a decrease in sensitivity. In addition, the clay becomes fissured. The use of electro-osmosis for altering the properties of clays in this manner has not been as frequent as that for the stabilization of slopes in silty materials.

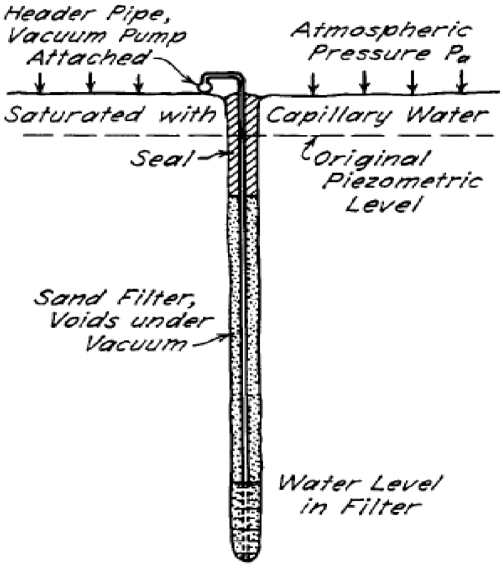


Fig: Vacuum tube

ELECTRO-OSMOSIS METHOD

The principle of this method has been earlier. It has most often been applied in practice to the stabilization of slopes being excavated into cohesion less or slightly cohesive silts below the normal groundwater level. The time required to drain such materials by the vacuum method may be excessive, especially under emergency conditions. Yet, the materials readily become quick under the influence of the seepage pressures directed inward toward the face and upward toward the bottom of the excavation. By an arrangement of electrodes similar to that shown in Fig., and the application of a suitable potential, seepage pressures due to electro-osmotic flow can be created in directions away from the faces of the excavation and toward the cathodes. The stabilizing influence of these pressures is in many instances spectacular and occurs as soon as the current is turned on. In addition there are a progressive decrease in the water content of the silt and a corresponding increase in strength (Casagrande 1949, 1962).

The anodes commonly consist of iron pipes, although reinforcing bars or steel rails have also been used. Corrosion is likely to be concentrated at a few points of the anodes; consequently the anodes may become discontinuous whereupon the lower portions are no longer effective. If the anodes consist of pipes, smaller pipes or rods can be inserted in them to restore their continuity. The cathodes may consist merely of iron rods along which the water flows as it escapes to the surface, but should preferably be perforated pipes screened for their full length to permit easier and more rapid escape of the water. The applied potential is usually on the order of 100 volts; the current required for stabilization of even a fairly small excavation is likely to be at least 150 amps. The actual power requirements depend on the resistivity of the soil and vary considerably. Potential gradients in excess of about 50 volt may lead to excessive energy loss in the form of heat Electro-osmosis causes consolidation of compressible soils such as clays. The consolidation is accompanied by the original water table and the subgrade level. As often as a sample is taken, the water should be allowed to rise in the casing, and the elevation to which it rises should appear in the boring record. Excavations in soils with high permeability *(*k greater than *ds)* or in very dense mixed-grained soils of medium permeability (k between and *ds)* as a rule can be drained without undue risk by pumping from open sumps. Under favorable conditions uniform soils of medium permeability can also be drained without mishap by pumping from sumps. However, this procedure involves the possibility of the formation of boils on the bottom of the excavation, associated with underground erosion and subsidence of the area surrounding the excavation. To avoid this risk it is preferable to drain soils of medium permeability by pumping from well points or filter wells. The drainage of the soil prior to excavation requires 2 to 6 days. The greatest depth to which the water table can be lowered by drawing the water from one set of wells or well points is about 6m. If the bottom of the proposed excavation is located at a greater depth, a multiple-stage setup may be used. Two or more header pipes must be installed at a vertical spacing not exceeding about 5m. If limitations of space do not permit a multiple-stage installation, eductor well points may be suitable. If the depth of the excavation exceeds about 15m, it is usually preferable to drain the soil adjoining the site by means of deep-well pumps operating within the casings of large diameter wells.

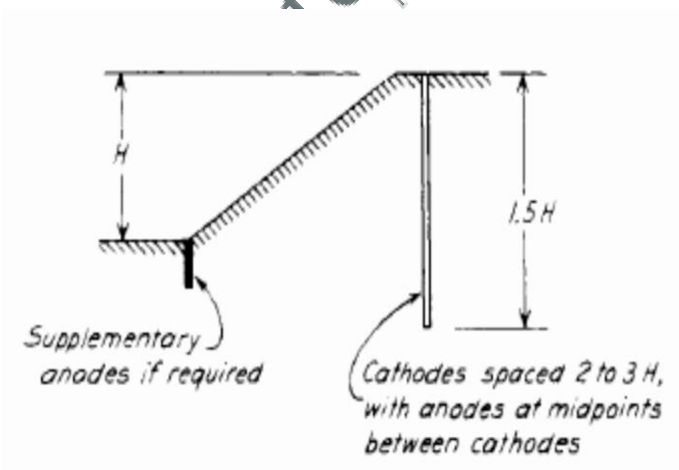


Fig: Elecro- osmosis

1.11 RECOMMENDED QUESTIONS

1. What are the objectives of subsurface exploration?
2. Describe with a neat sketch wash boring method of soil exploration.
3. List and explain various types of samplers
4. Explain seismic refraction method of soil exploration with neat sketch.
5. List out the methods of dewatering. Explain Vacuum method of dewatering with neat sketch
6. In a seismic survey the following readings were obtained

Time(S)	0.1	0.2	0.3	0.4	0.45	0.5	0.55
Distance (m)	40	80	120	160	200	240	280

Geophones are fixed at 40 m in a straight line. Determine:

i) Wave velocity in soil layers
ii) Thickness of top stratum

7. Estimate the ground water level by Hvorslev's method using the data given. Depth up to which water is bailed out is 30m, rise in water level after first day is 2.2m, second day 1.8m and on third day it is 1.5m.
8. A sampling tube has inner diameter of 70mm and cutting edge of 68mm. its outside diameters are 72 mm and 74mm respectively. Determine area ratio, inside clearance, outside clearance of the sampler. This tube is pushed at the bottom of the borehole to a distance of 580mm with length of sample recorded being 520mm. find the recovery ratio.

1.12 OUTCOMES

Students should be able to

- Decide upon soil exploration techniques to be adopted for different site condition
- Conduct soil exploration and to do the report of the same
- Collect soil sample by using proper sampling technique base on requirement
- Understand dewatering techniques and efficiency of lowering of water table

1.13 FURTHER READING

- http://www.yourarticlelibrary.com/soil/soil-exploration-purpose-planning-investigation-and-tests/45862/
- https://theconstructor.org/geotechnical/soil-investigation-and-exploration/2411/

MODULE 2
STRESSES IN SOIL

Structure

2.0 Objectives

- To determine magnitude of stresses in soils
- To apply various theories and methods to determine stresses in soils
- To understand contact pressure and pressure distribution in soils
- To Estimate settlement in soils

2.1 Introduction

Stresses in soil are caused due to

a) Self weight of soil

b) Structural loads, applied at or below the surface

The estimation of vertical stresses at any point in a soil mass due to external loading is essential to the prediction of settlements of buildings, bridges and pressure.

The stresses induced in a soil due to applied loads depend upon its Stress – Strain characteristics. The stress strain behaviour of soils is extremely complex and it depend upon a large number of factors, such as drainage conditions, water content, void ratio, rate of loading, the load level, and the stress path. However simplifying assumptions are generally made in the analysis of soil behaviour to obtain stresses. It is generally assumed that the soil mass is homogeneous and isotropic. The stress strain relationship is assumed to be linear. The theory of elasticity is used to determine the stresses in the soil mass. Though it involves considerable simplification of real soil behaviour and the stresses computed are approximate, the results are good enough for soil problems usually encountered in the practice.

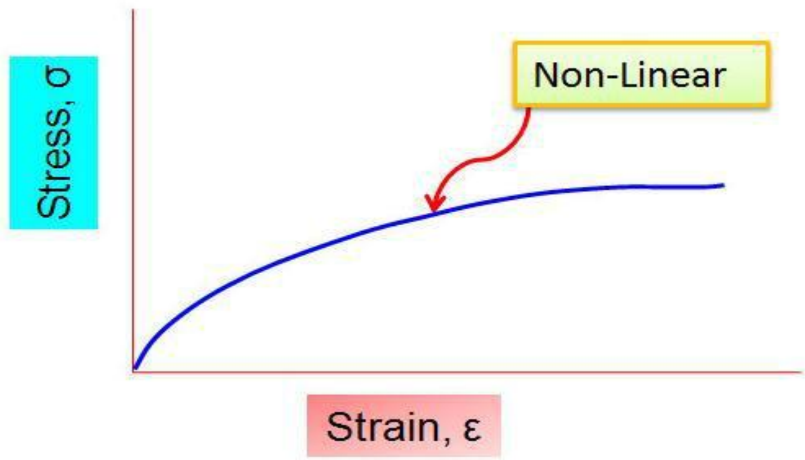


Fig.2.1 Typical Stress-strain curve for soils

2.2 Boussinesq's Theory / Solution

Boussinesq (1885) has given the solution for the stresses caused by the application of the point load at the surface of a elastic medium with the aid of the mathematical theory of elasticity.

3.3.1 Assumptions

1) The soil medium is an elastic continuum having a constant value of modulus of elasticity (E). i.e. it obeys Hooke"s law
2) The soil is homogeneous, i.e. it has identical elastic properties at all points in identical directions.
3) The soil is isotropic, i.e. it has identical elastic properties in all direction at a point.
4) The soil mass is semi-infinite, i.e. it extends to infinity in the downward directions and lateral directions. In other words, it is limited on its top but a horizontal plane and extends to infinity in all other directions.

5) The self weight of the soil is ignored.
6) The soil is initially unstressed i.e. it is free from residual stresses before the application of the load.
7) The top surface of the medium is free of shear stresses and is subjected to only the point load at a specified location.
8) Continuity of stress is considered to exist in the medium.
9) The stresses are distributed symmetrically with respect to z axis (vertical axis).

2.3 Vertical stresses due to a concentrated load

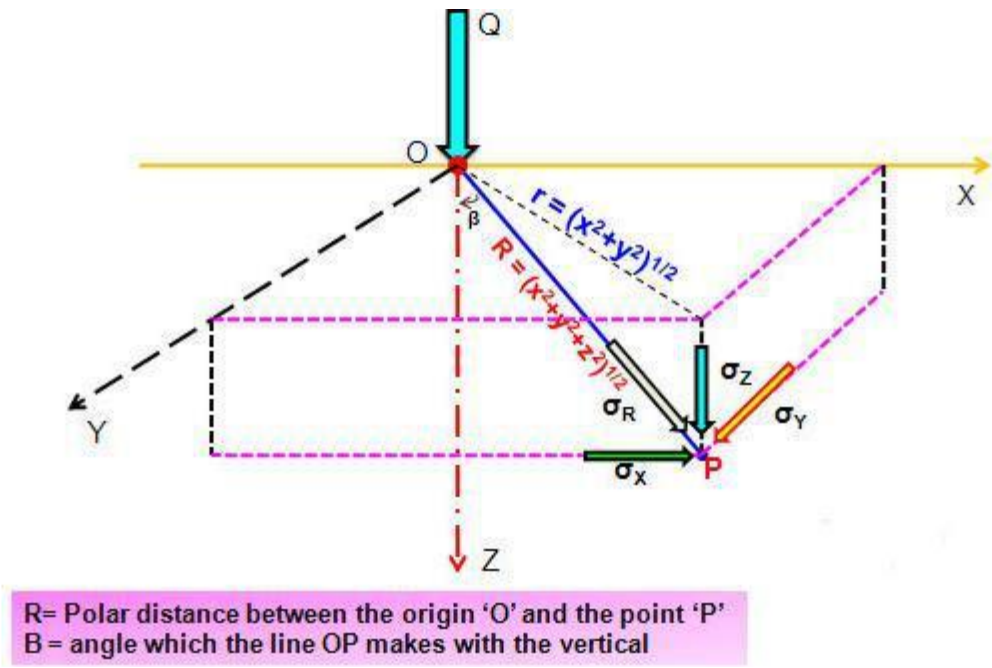


Fig.2.2 Derivation of vertical stress at a point due to point load acting on the ground surface

Figure 3.2 shows a horizontal surface of an elastic continuum subject to a point load Q at a point O. Using logarithmic stress function for solution of elasticity problem, Bossinesq proved that the polar stress σ_R at point P is given by

$$\sigma_R = \frac{3}{2\pi}\left(\frac{Q\cos\beta}{R^2}\right) \quad \text{---------(3.1)}$$

where R = Polar distance between the origin O and point t

β = angle which the line OP makes with the vertical or $R = \sqrt{r^2 + Z^2}$

Where $r^2 = x^2 + y^2$

$\text{Sin}\beta = \frac{r}{R}$ and $\cos\beta = \frac{Z}{R}$

Now, the vertical stress(σ_z)at Point P is given by

$\sigma_z = \sigma_R \cos^2\beta$ -------------(3.2)

$= \frac{3}{2\pi}(\frac{Q\cos\beta}{R^2})\cos^2\beta$

$= \frac{3Q}{2\pi}(\frac{\cos^3\beta}{R^2})$

$= \frac{3Q}{2\pi}(\frac{(\frac{Z}{R})^3}{R^2})$

$\boxed{\sigma_z = \frac{3Q}{2\pi}(\frac{Z^3}{R^5})}$ ------------------------(3.3)

$= \frac{3Q}{2\pi}(\frac{Z^2}{Z^2}\frac{Z^3}{R^5})$

$= \frac{3Q}{2\pi}(\frac{Z^5}{Z^2 R^5})$

$\sigma_z = \frac{3Q}{2\pi}\frac{1}{Z^2}(\frac{Z^5}{(r^2+Z^2)^{\frac{5}{2}}})$

$$\boldsymbol{\sigma_z \frac{3Q}{2\pi}\frac{1}{Z^2}\left(\frac{1}{(1+(\frac{r}{z})^2)^{\frac{5}{2}}}\right)} \text{ ----------------------- } \mathbf{(3.4)}$$

or, $\boldsymbol{\sigma_z = \frac{I_B Q}{Z^2}}$ --------------------------------- **(3.5)**

Where $I_B = \frac{3}{2\pi}(\frac{1}{(1+(\frac{r}{z})^2)^{\frac{5}{2}}})$ ----------------------(3.6)

The coefficient I_B is known as the Boussinesq influence for the vertical stress. The value of I_B can be determined for the given value of r/Z from Eq. 3.6.

$$\sigma_x = \frac{Q}{2\pi}\left[\left(\frac{3x^2Z}{R^5} - (1-2\nu)\left(\frac{x^2-y^2}{R\,r^2(R+Z)} + \frac{y^2Z}{R^3r^2}\right)\right]\text{------------ (3.7)}$$

$$\sigma_y = \frac{Q}{2\pi}\left[\left(\frac{3y^2Z}{R^5} - (1-2\nu)\left(\frac{y^2-x^2}{R\,r^2(R+Z)} + \frac{x^2Z}{R^3r^2}\right)\right]\text{------------(3.8)}$$

If ν = 0.5

$$\boxed{\sigma_x = \frac{3Q}{2\pi}\frac{x^2Z}{R^5}} \qquad \boxed{\sigma_y = \frac{3Q}{2\pi}\frac{y^2Z}{R^5}}$$

$$\sigma_y = \frac{Q}{2\pi}\left[\left(\frac{3y^2Z}{R^5} - (1-2\nu)\left(\frac{y^2-x^2}{R\,r^2(R+Z)} + \frac{x^2Z}{R^3r^2}\right)\right]\text{------------(3.8)}$$

If ν = 0.5

$$\boxed{\sigma_x = \frac{3Q}{2\pi}\frac{x^2Z}{R^5}} \qquad \boxed{\sigma_y = \frac{3Q}{2\pi}\frac{y^2Z}{R^5}}$$

The following points are to be noted while using Eq. (3.5)

1) The vertical stress does not depend upon the modulus of elasticity (E) and the poisson ratio (ν). But the solution has been derived assuming that the soil is linearly elastic.
2) The Intensity of vertical stress just below the load point is given by

   $\sigma_Z = 0.4775\frac{Q}{Z^2}$ (∵ when, r/z = 0, I_B = 0.4775)
3) At the surface (z = 0), the vertical stress just below the load is theoretically infinite. However, in an actual case, the soil under the load yields due to very high stresses. The load point spreads over a small but finite area and, therefore, only finite stresses develop.
4) The vertical stress (σ_Z) decreases rapidly with an increase in r/z ratio.

   Theoretically, the vertical stress would be zero only at an infinite distance from the koad point. Actually, at r/z = 5, or more, the vertical stress becomes extremely small and is neglected.

5) In actual practice, foundation loads are not applied directly on the ground surface. However, it has been established that the Boussinesq‟ s solution can be applied conservatively to field problems concerning loads at shallow depths, provided the distance „z‟ is measured from the point of application of the load.
6) Boussinesq‟ s solution can even be used for negative (upward) loads. For example if the vertical stress decrease due to an excavation is required, the negative load is equal to the weight of the soil removed. However, as the soil is not fully elastic the stresses determined are (necessarily) approximate.
7) The field measurements indicate that the actual stresses are generally smaller than the theoretical values given by Boussinesq‟ s solution especially at shallow depths. Thus, the Boussinesq‟ s solution gives conservative values and is commonly used in soil engineering problems.

Limitations

1) The solution was derived assuming the soil as an elastic medium, but the soil doesnot behave as an elastic material.
2) When the stress decrease occurs in soil, the relation between the stress and the strain is not linear as assumed, therefore, the solution is not strictly applicable.
3) In deep sand deposits, the modulus of elasticity increases with an increase in depth and therefore, the Boussinesq‟s solution will not give satisfactory results.
4) The point loads applied below ground surface causes somewhat smaller stresses than are caused by surface loads, and, therefore, the solution is not strictly applicable.

Numerical Example

1) Find intensity of vertical pressure at a point 3 m directly below 25 kN point load acting on a horizontal ground surface. What will be the vertical pressure at a point 2m horizontally away from the axis of loading and at same depth of 3 m? Use Boussinesq‟s equation.

Solution: Case (i)

Here, Q = 25 kN

z = 3 m

r = 0

$$\sigma_z = \frac{3}{2\pi}\frac{Q}{z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{\frac{5}{2}}$$

$$= 1.33\ kN/m^2$$

Case (ii)

Here,

r = 2 m

$\sigma_z = 0.53\, kN/m^2$

2.4 Stress Isobar or pressure bulb

An „isobar" is a stress contour or a curve which connects all points below the ground surface at which the vertical pressure is the same. An isobar is a spatial curved surface and resembles a bulb in shape. The stress isobar is also called „pressure bulb".

Any number of pressure bulb may be drawn for any applied load, since each one corresponds to an arbitrarily chosen value of stress. The isobar of a particular intensity can be obtained by:

$$\sigma_z = \frac{I_B\, Q}{Z^2}$$

An isobar consisting of a system of isobars appears somewhat as shown in Fig.

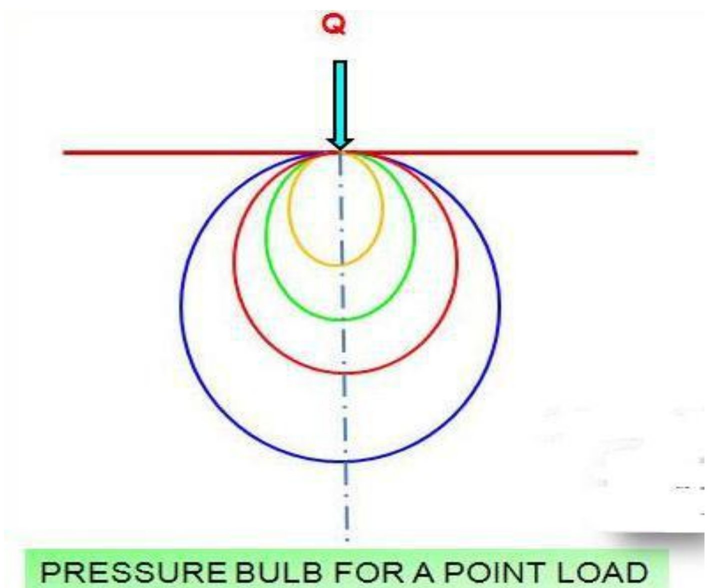


Fig. 2.3 Pressure bulb for a point load

2.5.1 Procedure

The procedure for plotting an isobar is as follows:

Let it be required to plot an isobar for which $_z$ = 0.1Q per unit area (10 % isobar)

$$0.1Q = \frac{I_B\,Q}{Z^2}$$

$$I_B = 0.1Z^2$$

$$\sigma_Z = \frac{I_B\,Q}{Z^2}$$

We know that

Assuming various values of z, the corresponding I_B values are computed. For the values of I_B, the corresponding r/z values are determined and hence the values of r are obtained.

An isobar is symmetrical about the load axis, the other half can be drawn from symmetry.
When r = 0, I_B = 0.4775; the isobar crosses the line of action of the load at a depth of

$$Z = \sqrt{\frac{I_B}{0.1}} = \sqrt{\frac{0.4775}{0.1}} = \sqrt{4.775} = 2.185 \text{ units}$$

The shape of an isobar approaches a lemniscates curve (not circle) as shown in Fig.3.3

The calculations are best performed in the form of a table as given below:

When r = 0 $\quad I_B = \frac{3}{2\pi}\left(\frac{1}{(1+(\frac{r}{z})^2)^{\frac{5}{2}}}\right)$

$$= \frac{3}{2\pi}$$

$$= 0.4775$$

Table 3.1 Data for isobar of $_z$ = 0.1Q per unit area

Depth Z (units)	Influence coefficient)*	r/Z	r (units)	$_z$
0.5	0.0250	1.501	0.750	0.1Q
1.0	0.1000	0.932	0.932	0.1Q
1.5	0.2550	0.593	0.890	0.1Q
2.0	0.4000	0.271	0.542	0.1Q
2.185	0.4775	0	0	0.1Q

2.5 Pressure distribution

It is possible to calculate the following pressure distribution by Eq.3.5 of Boussinesq and present them graphically

2.5.1 Vertical pressure distribution on horizontal plane

The vertical stress on horizontal plane at depth „z" is given by

$$\sigma_Z = \frac{I_B\, Q}{Z^2}$$

Z being a specified depth

For several assumed values of r, r/Z is calculated and the influence factor I_B, is found for each, the value of is then computed.

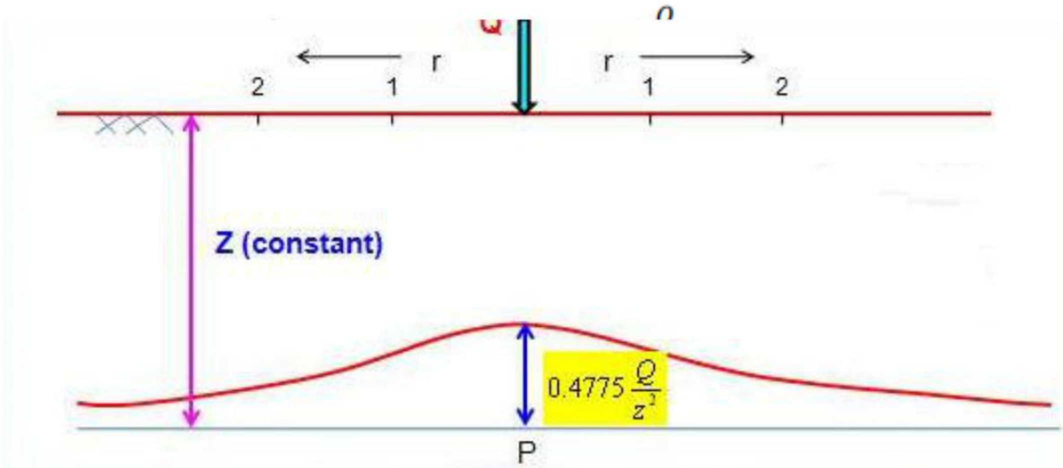


Fig.2.4 Vertical pressure distribution on horizontal plane

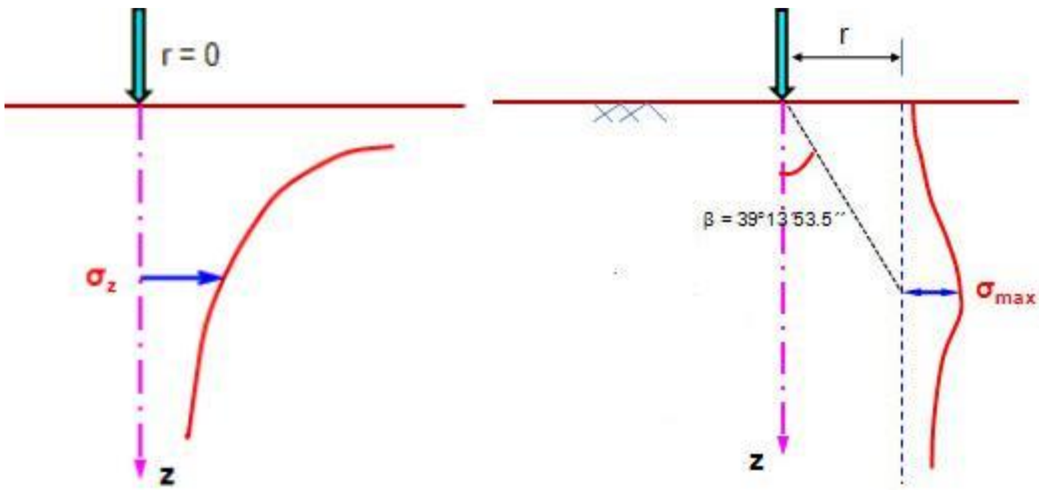
Theoretically, the stress approaches zero at infinity. However, practically it reaches a negligible value at a short finite distance. The maximum pressure ordinate is relatively high at shallow elevations and it decreases with increase depth. In other words, the bell shaped fig flattens out with increasing depth. The vertical stress distribution diagram on a horizontal plane can also be obtained graphically if the isobars of different intensities are available. The horizontal plane is drawn on the isobar diagram. The points of intersection of the horizontal plane with the isobar of a particular intensity give that vertical stress.

2.5.2 Vertical stress distribution on a vertical plane

The vertical stress distribution on a vertical plane at a radial distance of „r‟ can be obtained from

$$\sigma_Z = \frac{I_B\, Q}{Z^2}$$

The variation of vertical stress with depth at a constant radial distance from the axis of the load is as shown in Fig.3.5



a) directly under the point load **b) at a distance from point load**

Fig.2.5 Vertical pressure distribution on vertical plane

In this case radial distance „r‟ is constant and the depth „z‟ changes. As z increases, r/z decreases, for a constant value of „r‟. As r/z decreases +, value in the equation for $_z$ increases, but, since E is involved in the denominator of the expression for $_Z$, its value first increases with depth, attains a maximum value, and then decreases with further increase in depth.

The maximum vertical stress occur at r/Z=0.817.this corresponds to the point of intersection of the vertical plane with the line drawn at $39^0$13‟53.5” to the vertical axis of the load.

2.6 Vertical stress due to a line load

Certain road and rail traffic loads, and loads from walls may be resolved into line load, which has length along a given line but not breadth (theoretical)

The vertical stress in a soil mass due to a vertical line load can be obtained using boussinesq‟s solution. Let the vertical line load be of intensity q per unit length, along the y axis, acting on the surface of a semi-infinite soil mass as shown in Fig.

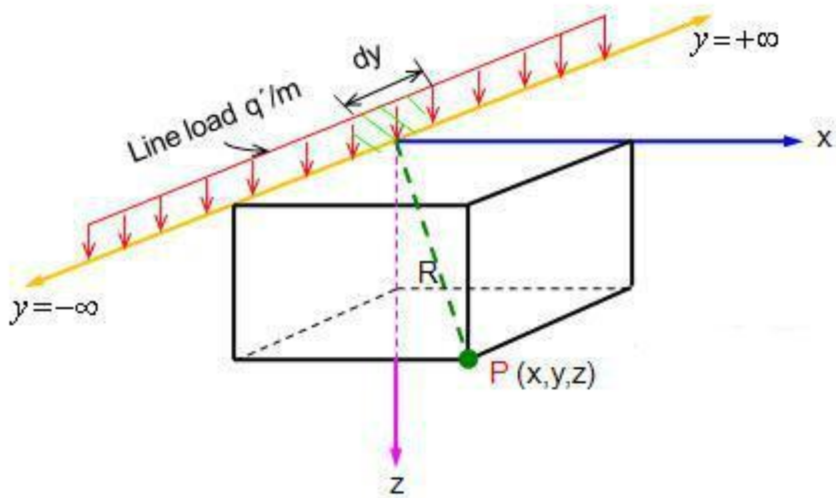


Fig.2.6 Vertical stress due to a line load

Let us consider the load acting on a small length dy. The load can be taken as a point load of q‟ dy. using boussinesq‟s solution the vertical stress at P is given by

$$\Delta\sigma_Z = \frac{3(q' dy)}{2\pi} \frac{Z^3}{(r^2+Z^2)^{\frac{5}{2}}}$$ --------------------------(3.9)

$$\therefore \sigma_Z = \frac{3Q}{2\pi}\frac{1}{Z^2}\left(\frac{Z^5}{(r^2+Z^2)^{\frac{5}{2}}}\right)$$

The vertical stress at P due to line load extending from -∞ to + ∞ is obtained by integration,

$$\sigma_Z = \frac{3q'Z^3}{2\pi}\int_{-\infty}^{\infty}\frac{dy}{(r^2+Z^2)^{\frac{5}{2}}}$$

$$\sigma_Z = \frac{3q'Z^3}{2\pi}\int_{-\infty}^{\infty}\frac{dy}{(x^2+y^2+Z^2)^{\frac{5}{2}}}$$ ------------(3.10)

Substituting $x^2 + Z^2 = u^2$ in Eq. 3.10 we get

$$\sigma_Z = \frac{3q'Z^3}{2\pi}\int_{-\infty}^{\infty}\frac{dy}{(u^2+y^2)^{\frac{5}{2}}}$$ -----------------(3.11)

Let $y = u \tan\theta$ $(dy = u \sec^2 \theta . d\theta)$

When, $y = -\infty$, $\tan\theta = -\infty$, $\theta = -\pi/2$

$Y = +\infty$, $\tan\theta = \infty$, $\theta = \pi/2$

Eqn (3.11) can be written as

$$\sigma_Z = \frac{3q'Z^3}{2\pi}\int_{0}^{\pi/2}\frac{u\sec^2\theta\, d\theta}{(u^2+u^2\tan^2\theta)^{\frac{5}{2}}}$$

$$\sigma_Z = \frac{3q'Z^3}{2\pi}\int_{0}^{\pi/2}\frac{u\sec^2\theta\, d\theta}{u^5\sec^5\theta}$$ $\{\because (1+\tan^2\theta) = \sec^2\theta\}$

$$\sigma_Z = \frac{3q'Z^3}{2\pi u^4}\int_{0}^{\pi/2}\cos^3\theta . d\theta$$ -------------------(3.12)

Let $\sin\theta = t$

$\cos\theta . d\theta = dt$

When $\theta = 0$, $\sin\theta = 0$

$\theta = \pi/2$, $\sin\theta = 1$

Also $\cos^3\theta d\theta = \cos^2\theta . \cos\theta . d\theta$

$\cos^2\theta + \sin^2\theta = 1$

or $\cos^2\theta = (1-\sin^2\theta)$

$\therefore \cos^3\theta\, d\theta = (1 - \sin^2\theta) \cos\theta\, d\theta$

(3.12) becomes,

$$\sigma_Z = \frac{3q'Z^3}{\pi u^4}\int_0^1 (1-t^2).\,dt$$

$$= \frac{3q'Z^3}{\pi u^4}\ \left(t - \frac{1}{3}t^3\right)_0^1$$

$$\sigma_Z = \frac{3q'Z^3}{\pi u^4}\frac{2}{3}$$

$$= \frac{2q'Z^3}{\pi u^4}$$

$$= \frac{2q'Z^3}{\pi(x^2+Z^2)^2}$$

$$\boldsymbol{\sigma_Z = \frac{2q'}{\pi Z}\left(\frac{1}{1+(\frac{x}{z})^2}\right)^2} \text{ ---------------------------- } \mathbf{(3.13)}$$

$$\sigma_Z = (I_B)_L \frac{q'}{Z}$$

$(I_B)_L$ = Boussinesq's influence factor for line load

$$= \frac{2}{\pi}\left(\frac{1}{1+(\frac{x}{z})^2}\right)^2$$

When the point P lies vertically below the line load x = 0

$$\sigma_Z = \frac{2q'}{\pi Z}$$

Numerical Example

2) A line load of 100 kN/m run extends to a long distance. Determine the intensity of vertical stress at a point, 2m below the surface for the following two cases:

i) Directly under the line load, and

ii) At a distance of 2 m perpendicular to the line load.

Use Boussinesq‟s theory

Solution:

Data:

q =100 kN/m z = 2 m

Using Boussinesq‟ s theory, the stress at any depth „z‟ for line load is given by:

$$\sigma_z = \frac{2q'}{\pi z}\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2$$

Case (i)

x = 0

∴ σ_Z = 31.83 kN/m^2

Case (ii)

x = 2 m; z = 2; x/z = 1

∴ σ_Z = 7.96 kN/m^2

2.7 Vertical stress due to a Strip load

The expression for vertical stress at any point „P‟ under a strip load can be derived from Eq. (2.13) of the line load. The expression will depend upon whether the point P lies below the centre of the strip load or away from the centre.

2.7.1 Point P below the centre of the strip

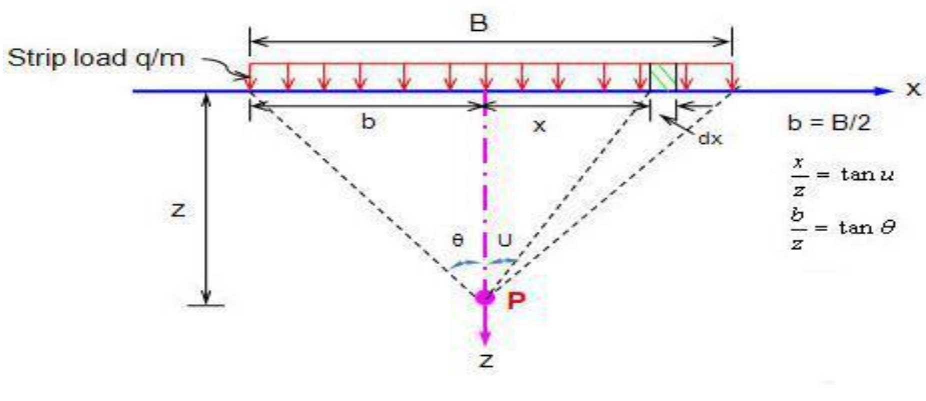


Fig. 2.7 Vertical stress due to a strip load- point 'P' below the centre

Consider a strip load of width B (= 2b) and intensity „q" as shown in the Fig.3.7

Let us consider the load acting on a small elementary width dx at a distance x from the center of the load. This small load of (q dx) can be considered as a line load of intensity q.

From Eq. 2.13, we have

$$\Delta\sigma_Z = \frac{2q'}{\pi Z}\left(\frac{1}{1+(\frac{x}{z})^2}\right)^2 \quad \text{where } q' = q\, dx$$

The stress due to entire strip load is obtained as

$$\sigma_Z = \frac{2q'}{\pi Z}\int_{-b}^{+b}\left(\frac{1}{1+(\frac{x}{z})^2}\right)^2 .dx \qquad \text{--------------------(3.14)}$$

Let $\frac{x}{Z} = tanu$, dx = $zsec^2u\ du$

$$\sigma_Z = \frac{2q}{\pi Z} 2\int_0^{\pi} \frac{z\ sec^2u}{(1+tan^2u)^2} .du$$

Where, $\theta = \tan^{-1}\frac{b}{z}$

$$\sigma_Z = \frac{4q}{\pi}\int_0^{\theta} cos^2u\ .du$$

$$= \frac{4q}{\pi}\int_0^{\theta}\frac{1+cos2u}{z} .du$$

$$= \frac{2q}{\pi}\left[u + \frac{sin2u}{2}\right]_0^{\theta}$$

$$= \frac{2q}{\pi}\left[\theta + \frac{sin2\theta}{2}\right]$$

$$\boldsymbol{\sigma_Z = \frac{q}{\pi}[2\theta + sin2\theta]} \qquad \text{-------------------------- (3.15)}$$

2.7.2 Point P not below the centre of the strip

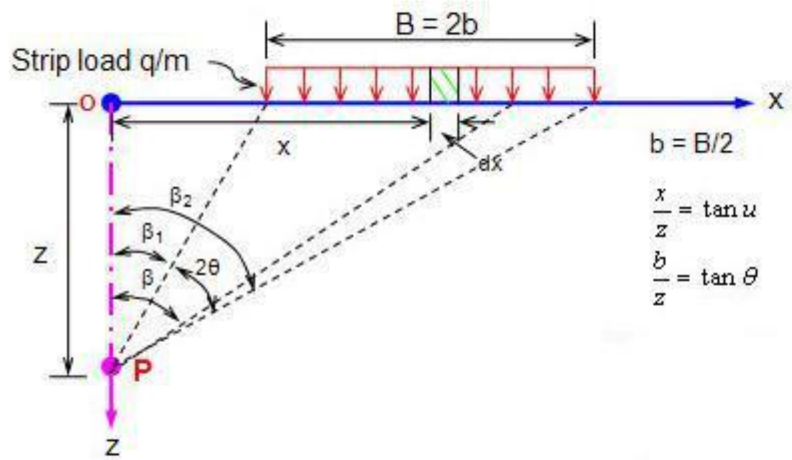


Fig.2.8 Vertical stress due to a strip load- point 'P' not at the centre

Fig. 3.8 shows the case when the point P is not below the centre of the strip. Let the extremities of the strip make angles of s and s with the vertical at P. Similar to the previous case, the load q dx acting on a small length dx can be considered as a line load. The Vertical stress at P given by Eq. 2.13 as

$$\Delta\sigma_Z = \frac{2(qdx)}{\pi Z}\left(\frac{1}{1+(\frac{x}{z})^2}\right)^2$$

$$Now\ substituting\ x = Z\ tan\beta \quad \mathrm{dx} = \mathrm{z}\ sec^2\beta d\beta$$

$$\Delta\sigma_Z = \frac{2q(Zsec^2\beta d\beta)}{\pi Z}\left(\frac{1}{1+tan^2\beta}\right)^2$$

$$\Delta\sigma_Z = \frac{2q}{\pi}cos^2\beta d\beta$$

$$\sigma_Z = \frac{2q}{\pi}\int_{\beta_{21}}^{\beta_2}\frac{1+cos2\beta}{2}d\beta$$

$$\text{Or } \sigma_Z = \frac{q}{\pi}\left[\beta + \frac{sin2\beta}{2}\right]_{\beta_1}^{\beta_2}$$

$$\sigma_Z = \frac{q}{\pi}[\beta + \ sin\beta\ cos\beta]_{\beta_1}{}^{\beta_2}$$

$\sigma_Z = \frac{q}{\pi}((\beta_2 - \beta_1) + (sin\beta_2 cos\beta_2 - sin\beta_1 cos\beta_1)$ ∵ (sin2β = 2sinβ co

$substituting\ \beta_2 - \beta_1 = 2\theta$

$$\sigma_Z = \frac{q}{\pi}(2\theta + sin\beta_2 cos\beta_2 - sin\beta_1 cos\beta_1) \text{ -------------------- (3.16)}$$

If $\beta_2 + \beta_1 = 2\varphi$, it can be shown that

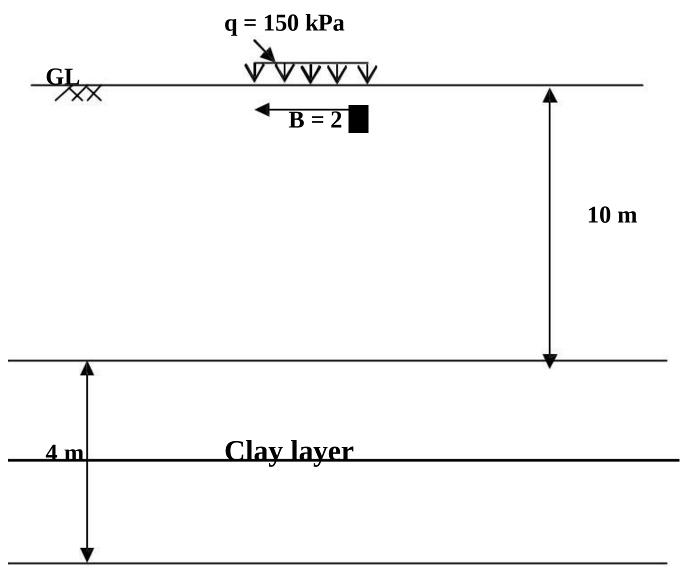
$sin\beta_2 cos\beta_2 - sin\beta_1 cos\beta_1 = sin2\theta\ cos2\emptyset$

$$\sigma_Z = \frac{q}{\pi}(2\theta + \ sin2\theta\ cos2\emptyset) \text{ -------------------------- (3.17)}$$

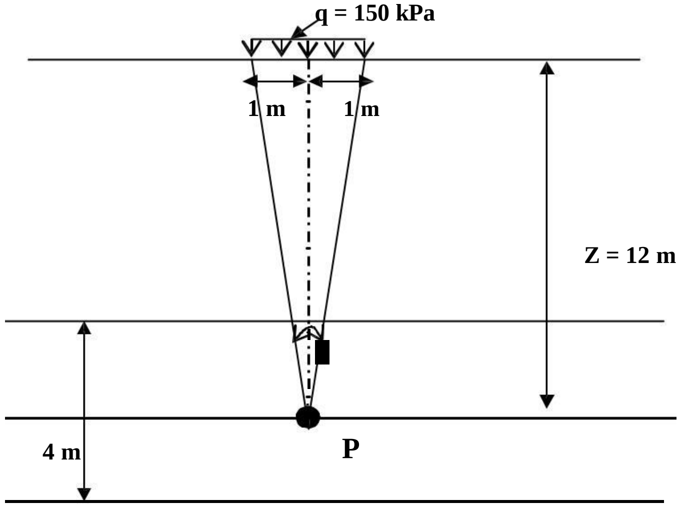
Numerical Example

3) **A strip footing 2 m wide is loaded on the ground surface with a pressure of 150 kN/m^2. A 4 m thick soft clay layer exists at a depth of 10 m below the foundation. Find the average increase in vertical stress at the centre of clay layer below the centre line and at the edge of footing. Adopt Boussinesq's theory for strip load.**

Solution:



Case (i): Point at the centre of clay layer below the centre line of strip load



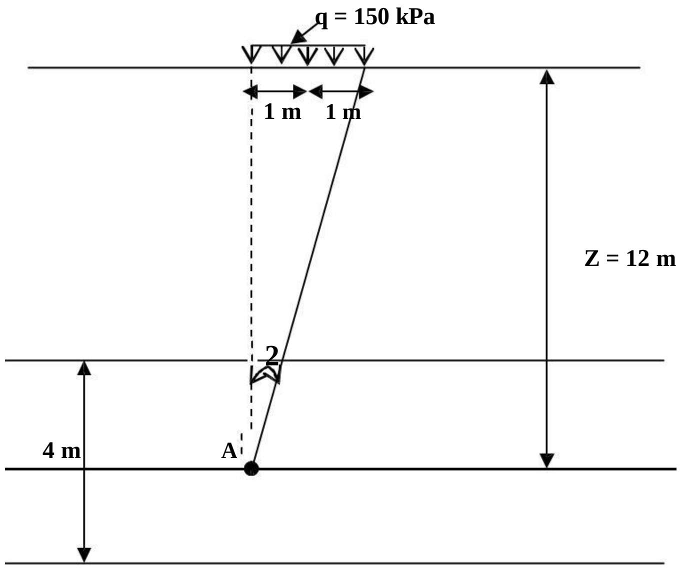
$\theta = \tan^{-1}(1/12) = 4°45\ 49$ = 0.083 rad

$\therefore 2\theta$ = 0.166 rad

$$\sigma_z = \frac{q}{\pi}[2\theta + \sin 2\theta]$$

or $\sigma_z = 15.83$ kN/m^2

Case (ii): Point at the centre of clay layer below the edge of footing (strip load)



$2\Theta = \tan^{-1}(2/12) = 9°27\ 44$ = 0.166 rad

$$\sigma_z = \frac{q}{\pi}[2\theta + \sin 2\theta]$$ = 15.72 kN/m^2

2.8 Vertical Stress under a uniform loaded circular area

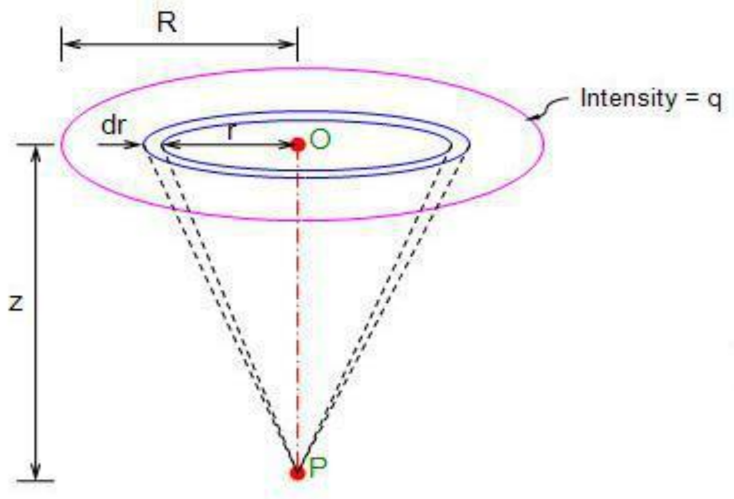


Fig.2.9 Vertical Stress under a uniform loaded circular area

This problem may arise in connection with settlement studies of structures on circular foundations, such as gasoline tanks, grain elevators, and storage bins.

The Boussinesq‟s equation for vertical stress due to a point load can be extended to find the vertical stress at any point beneath the centre of a uniformly loaded circular area.

Let q = intensity of the load per unit area

And R = the radius of the loaded area

Let us consider an elementary ring of radius r and thickness „dr‟ of the loaded area.

The load on the elementary ring = q(2πr)dr

But we know that

$$\sigma_Z = \frac{3Q}{2\pi}\frac{1}{Z^2}\left(\frac{1}{(1+(\frac{r}{Z})^2)^{\frac{5}{2}}}\right)$$

$$\Delta\sigma_Z = \frac{3(q2\pi rdr)}{2\pi}\frac{1}{Z^2}\left(\frac{1}{(1+(\frac{r}{Z})^2)^{\frac{5}{2}}}\right)$$

$$\Delta\sigma_Z = \frac{3qrdr}{(r^2+Z^2)^{\frac{5}{2}}}Z^3$$

The vertical stress due to full load is given by

$$\sigma_z = 3\, q\, z^3 \int_0^R \frac{r\, dr}{(r^2+Z^2)^{\frac{5}{2}}} \quad \text{-----------------(3.18)}$$

$Let\ r^2 + Z^2 = u^2$	when r = 0, u = Z^2
$2r\, dr = du$	r = R, u = $R^2 + Z^2$

Eq. 3.18 becomes,

$$\sigma_z = 3\, q\, z^3 \int_{Z^2}^{R^2+Z^2} \frac{du}{2(u)^{\frac{5}{2}}}$$

$$\sigma_z = 3\, q\, z^3 \left(\frac{1}{\left(\frac{-3}{2}\right)}\right) \left[u^{\frac{-3}{2}}\right]_{Z^2}^{R^2+Z^2}$$

$$= -qz^3 \left[\frac{1}{(R^2+Z^2)^{3/2}} - \frac{1}{(Z^2)^{3/2}}\right]$$

$$= qz^3 \left[\frac{1}{(Z^2)^{3/2}} - \frac{1}{(R^2+Z^2)^{3/2}}\right]$$

$$\boldsymbol{\sigma_z = \left[1 - \left(\frac{1}{1+\left(\frac{R}{Z}\right)^2}\right)^{\frac{3}{2}}\right]} \quad \text{------------------------- (3.19)}$$

$$\sigma_z = I_c\, q \quad \text{------------------------(3.20)}$$

Where, I_C is the influence coefficient for the circular area and is given by

$$I_c = \left[1 - \left(\frac{1}{1+\left(\frac{R}{Z}\right)^2}\right)^{\frac{3}{2}}\right] \quad \text{--------------------------(3.21)}$$

Eq. 3.21 for the influence coefficient I_C can be written in terms of the angle 2Ө subtended at point P by the load.

Now tanӨ = R/Z

$$I_c = \left[1 - \left(\frac{1}{1+(\tan)^2}\right)^{\frac{3}{2}}\right]$$

$$=(1-(\cos^2\theta)^{3/2}$$

$$I_c = 1 - \cos^3\theta \quad \text{------------ (3.22)}$$

Numerical Example

4) A circular area 6 m in diameter, carries uniformly distributed load of 10 kN/m^2. Determine the vertical stress at a depth of 2m, 4 m and 8 m. Plot the variation of vertical stress with depth. [VTU- July 2006 – 8 M]

Solution:

According to Boussinesq‟s theory for a circular loaded area, the vertical pressure at any point below the centre of the loading is given by:

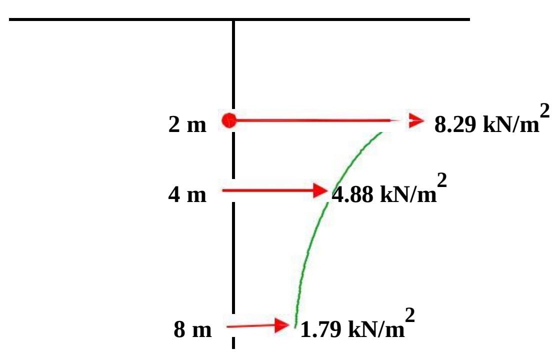
$$_z = q\left[1 - \left(\frac{1}{1 + \left(\frac{R}{z}\right)^2}\right)^{\frac{3}{2}}\right]$$

Here R = 3 m and q = 10 kN/m^2

At z = 2 m, $_z$ = 8.29 kN/m^2

At z = 4 m, $_z$ = 4.88 kN/m^2

At z = 8 m, $_z$ = 1.79 kN/m^2



2.9 Uniform load on rectangular area

The more common shape of a loaded area in foundation engineering practice is a rectangle, especially in the case of buildings.

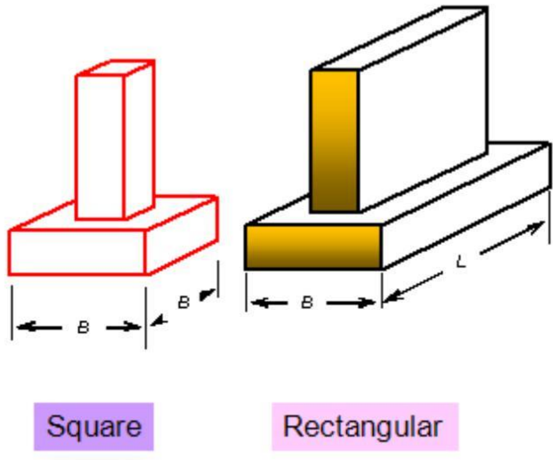


Fig. 2.10 Common shape of foundations

2.9.1 Vertical stress under corner of a uniformly loaded rectangular area

The vertical stresses under a corner of a rectangular area with a uniformly distributed load of intensity q can be obtained from Boussinesq"s solution.

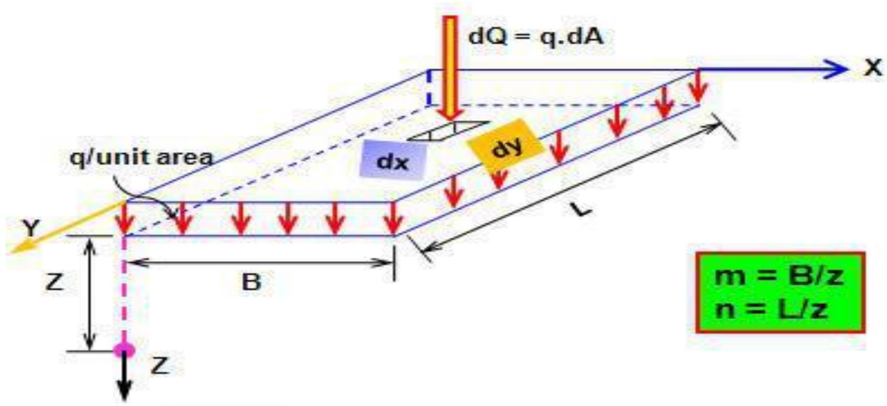


Fig. 2.11 Vertical stress under corner of a uniformly loaded rectangular area

$$\Delta\sigma_z = \frac{3(q \cdot dx \cdot dy) \times z^3}{2\pi} \frac{1}{(x^2 + y^2 + z^2)^{\frac{5}{2}}}$$

By integration,

$$\sigma_z = \frac{3qz^3}{2\pi} \int_0^L \int_0^B \frac{q \cdot dx \cdot dy}{\left(x^2 + y^2 + z^2\right)^{\frac{5}{2}}} \quad \text{--------------(3.23)}$$

Newmark (1935) has done the integration which is quite complicated and has expressed in the form:

$$\sigma_z = \frac{q}{4\pi}\left[\frac{2mn\sqrt{\left(m^2+n^2+1\right)}}{m^2+n^2+1+m^2n^2}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\frac{2mn\sqrt{\left(m^2+n^2+1\right)}}{m^2+n^2+1-m^2n^2}\right] \text{------(3.24)}$$

Where m = (B/Z) and n = (L/Z)

or $\sigma_z = I_N \, q_N$ ----------------------(3.25)

Where I_N is Newmark‟ s influence coefficient

Note: The value of m and n can be interchanged without any effect on the value of ‹Œ

Vertical stress at any point under a rectangular area

The equation as developed above can also be used for finding the vertical stress at a point which is not loaded below the corner of loaded area. The rectangular area is subdivided into rectangles such that each rectangle has a corner at the point where the vertical stress is determined using the principle of superposition.

The following cases can occur:

Point anywhere below the rectangular area

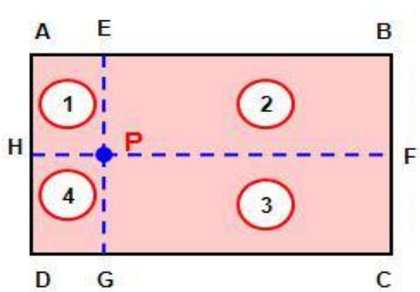


Fig. 3.12 Point anywhere below the rectangular area

Fig. 2.12 shows the location of the point P below the rectangular area ABCD. The given rectangle is subdivided into four small rectangles (AEPH, EBFP, HPGD and PFCG) each having one corner at p. the vertical stress at P due to the given rectangular load is equal to that from the four small rectangles.

$\sigma_Z = q\ [I_{N1} + I_{N2} + I_{N3} + I_{N4}]$

Where, I_{N1}, I_{N2}, I_{N3}, and I_{N4}, are Newmark‟s influence factors for the four rectangles marked 1, 2, 3, 4.

For the special case, when the point „P‟ is at the centre of the rectangle ABCD (Fig.2.13), all the four small rectangles are equal, and the above equation becomes

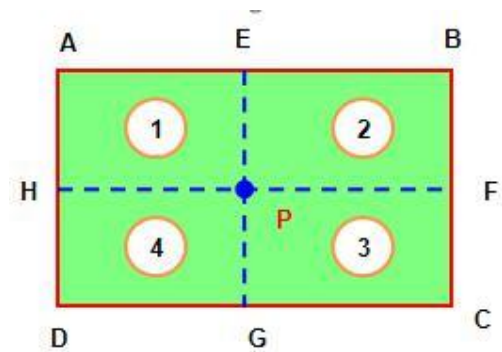


Fig. 2.13 Point at the centre of the rectangular area

$\sigma_Z = 4\ I_N\ q$

Where I_N = influence factor for the small rectangle.

Point outside the loaded rectangular area

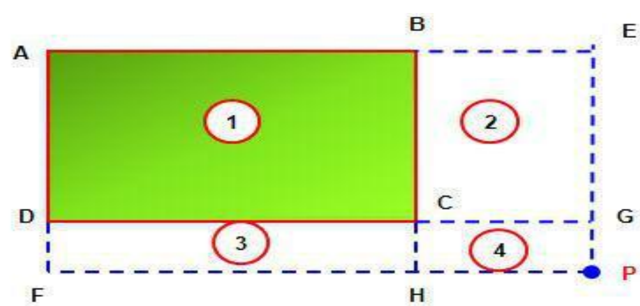


Fig. 2.14 Point outside the loaded rectangular area

Fig. 2.14 shows the point P outside the loaded area ABCD. In this case, a large rectangle AEPF is drawn with its one corner at P.

Now rectangle ABCD = rectangle AEPF – rectangle BEPH – rectangle DGPF + rectangle CGPH

The last rectangle CGPH is given plus sign because this area has been deducted twice (once in rectangle BEPH and once in DGPF)

The stress at P due to a load on rectangle ABCD is given by

$$\sigma_Z = q[(I_N)_1 - (I_N)_2 - (I_N)_3 + (I_N)_4]$$

Where, $(I_N)_1$, $(I_N)_2$, $(I_N)_3$, & $(I_N)_4$ are the influence coefficient for the rectangles AEPF, BEPH, DGPF, CGPH respectively.

Point below the edge of the loaded rectangular area

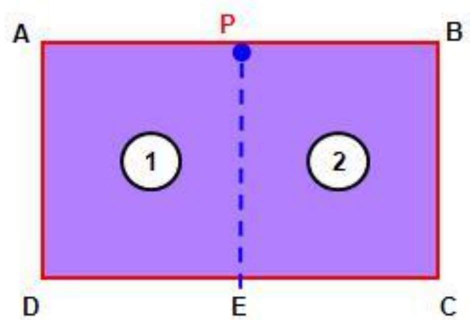


Fig. 2.15 Point below the edge of the loaded rectangular area

If the point P is below the edge of the loaded area ABCD (Fig. 2.15), the given rectangle is divided into two small rectangles APED and PBCE

$$\sigma_Z = q[(I_N)_1 + (I_N)_2]$$

Where, $(I_N)_1$ & $(I_N)_2$ are the influence coefficient for the rectangles 1 & 2.

2.10 Westergaard's Solution

Boussinesq‟s solution assumes that the soil deposit is isotropic. Actual sedimentary deposits are generally anisotropic. There are thin layers of sand embedded in homogeneous clay strata. Westergaard‟s solution assumes that there are thin sheets of rigid materials sandwiched in a homogeneous soil mass. These thin sheets are closely spaced and are of infinite rigidity and therefore prevent the medium from undergoing lateral strain. These permit only downward displacement of the soil mass as a whole without any lateral displacement. Therefore Westergaard‟s solution represents more closely the actual sedimentary deposits.

According to Westergaard the vertical stress at a point „P‟ at a depth „z‟ below the concentrated load Q is given by

$$\sigma_Z = \frac{C/2\pi}{\left[c^2 + \frac{r^2}{Z}\right]^{3/2}} \frac{Q}{Z^2} \text{-----------------------}(3.26)$$

Where, C depends upon the Poisson ratio (v) and is given by

$$C = \sqrt{\frac{(1-2v)}{(2-2v)}} \text{-----------------------}(3.27)$$

For an elastic material, the value of „*v'* varies between 0 to 0.5 Since it is assumed that there is lateral restrained, $v = 0$ & $C = 1/\sqrt{2}$

$$\sigma_Z = \frac{1}{\pi\left[1 + 2\frac{r^2}{Z}\right]^{3/2}} \frac{Q}{Z^2} = I_w \frac{Q}{Z^2} \text{----------------------}(3.28)$$

Where $I_w = \frac{1}{\pi\left[1 + 2\frac{r^2}{Z}\right]^{3/2}}$ is known as Westergaard's influence factor

2.11 Newmark's Influence Chart

2.11.1 Introduction:

The vertical stress at any particular depth in the soil due to the action of vertical load on the surface of the ground was given and explained by the famous Boussinesq‟s theory. This theory gave formulae to calculate vertical stresses at a point for different types of vertical loading, taking into consideration only a few well defined and standard shape of loading like a point loading, line loading, strip loading, rectangular loading and circular loading. When some complex shape of loading, like a plan of a structure was given, it became very cumbersome to calculate the vertical stress using these formulas. Hence, a need for more simpler and faster method of stress calculation was realized. Newmark formulated a new simple graphical method to calculate the vertical stress at any particular depth caused due to any shape of vertical uniformly distributed loading in the interior of an elastic, homogeneous and isotropic medium, which is bounded by horizontal planes (i.e. semi-infinite medium).

2.11.2 Theory behind construction of Newmark's Influence chart:

Newmark‟s chart utilizes the equation given by Boussinesq for vertical stress caused due to uniformly distributed load on a circular area (vertical load) at any particular depth.

$$\sigma_z = I_c q \qquad (2.20)$$

Where σ_z = vertical stress at depth of

q = load intensity

I_C = Boussinesq‟s influence value for circular loading.

The Boussinesq‟s influence factor, I_C depends on the radius of circular loaded area, R and on the depth, z, at which the stress is required. It is given by:

$$I_c = 1 - \frac{1}{\left[1 + (R/z)^2\right]^{\frac{3}{2}}}$$

From this equation the ratio (r/z) is expressed as:

$$\frac{R}{z} = \pm\sqrt{\left(1 - \frac{\sigma_z}{q}\right)^{-\frac{2}{3}} - 1} \qquad (2.29)$$

This ratio (r/z) represents the ***relative size or relative radii*** of circular loaded bearing area, which gives a particular value of vertical stress to applied load ration (σ_Z/q). The ratio (σ_Z/q) can vary from 0 to 1. By substituting various values of (σ_Z/q) with any desired interval (i) say 0.1 or 0.05 etc. we can get various relative size ratio i.e. (r/z). Now let us assign a series of values for the ratio (σ_Z/q), such as 0, 0.1, 0.2,…., 0.9, and 1. A corresponding set of values for the relative radii, (R/z) may be obtained. If a particular depth is specified, then a series of concentric circles can be drawn. Since the first circle has a zero radius and the eleventh has infinite radius. In practice, only nine circles are drawn. Each ring or annular space causes a stress of q/10 at a point beneath the center at the specified depth z, since the number of annular space (c) is ten. Thus, each loaded area enclosed between any two successive circles contributes the same influence on the vertical stress at the point we require. The typical tabulation of (R/z) and (σ_Z/q) values are shown with an interval, i = 0.1.

Table 2.2: Relative radii for Newmark's Influence chart

Sl. No. of circle	(σ_Z/q)	Relative radii (R/z)
1	0.0	0.000
2	0.1	0.270
3	0.2	0.400
4	0.3	0.518
5	0.4	0.637
6	0.5	0.766
7	0.6	0.918
8	0.7	1.110
9	0.8	1.387
10	0.9	1.908
11	1.0	∞

From this table it can be seen that the widths of the annular slices or rings are greater the farther away they are from the center. The circle for an influence of 1.0 has an infinitely large radius. Now let us assume that a set of equally spaced rays, say „*s'* in number, is drawn emanating from the center of the circles, thus dividing each annular area into *s* sectors, and the total number area into *cs* sectors. If the usual value of 20 is adopted for *s*, the total number of sectors in this case will be 10x20 or 200. Each sector will cause a vertical stress of $1/200^{th}$ of the total value at the center at the specified depth and is referred to as a „mesh‟ or an „influence unit‟. The value 1/200 or 0.005 is said to be the „influence value‟ or „influence factor‟ (I) for the chart. Each mesh may thus be understood to represent an influence area. The value of „c‟ and „s‟ are so selected that it gives the accuracy we desire for. Higher value of „c‟ and „s‟ gives more meshes and hence higher accuracy.

2.11.3 Construction of Newmark's Influence chart

For the specified depth z, say 10 m, the radii of the circles, R, are calculated from the relative radii of Table 1. (2.70m, 4.00m, 5.18m, …and so on). The circles are then drawn to a convenient scale (say, 1 cm = 2 m or 1:200). A suitable number of uniformly spaced rays (to get required influence value) are drawn, emanating from the center of the circles. The resulting diagram will appear as shown in Fig. 3.16. On the figure is drawn a line AB, representing the depth z to the scale used in drawing the circles. If the scale used is 1 cm = 2 m, then AB will be 5 cm. The influence value for this chart will be I = (1/c x s).

The same chart can be used for other values of the depth „z‟. The length AB is taken equal to the depth „z‟ of the given problem and to that scale the loaded diagram is plotted on a tracing sheet to be superimposed later on the Newmark‟s chart to obtain the vertical stress at the desired point.

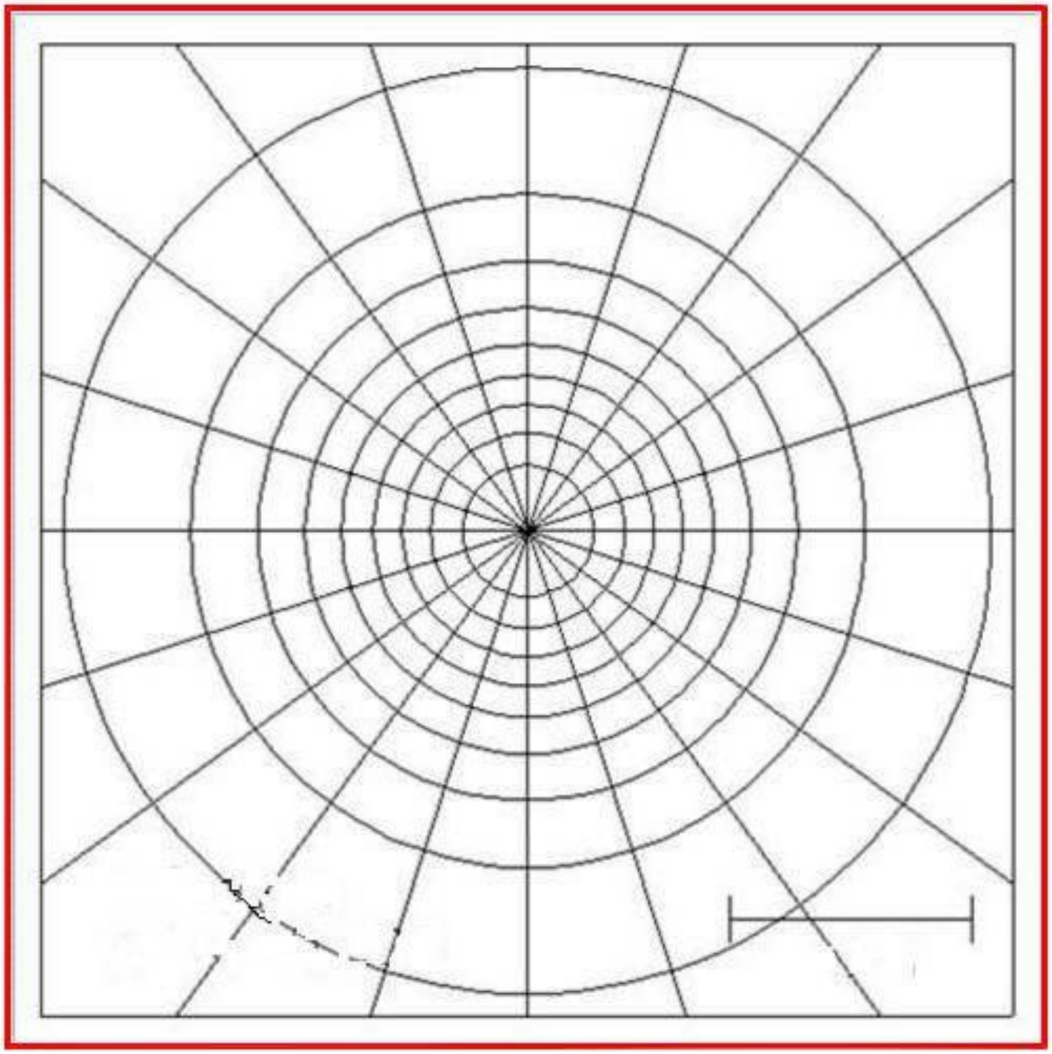


Fig. 2.16 Newmark's chart

2.11.4 Application of Newmark's Influence chart

Application of Newmark"s Influence chart in solving problems is quite easy and simple. The plan of the loaded area is first drawn on a tracing sheet to the same scale as the scale of the line segment AB on the chart representing the depth „z". The location of the point where the vertical stress is required is marked on the plan, say as „P". Now, the tracing sheet is placed over the chart, such that the point „P" comes exactly over the center of the chart from where the rays are emanating. Now the number of mesh covered by the plan is counted. In case of partly covered mesh an intelligent judgement of the fraction of mesh covered is required. Let the total number of mesh be equal to „n" . Then the vertical stress at the desired depth is given by:

$$\sigma_z = I \times n \times q$$

Where I = Influence value = $1/(c \times s)$

n = Number of meshes under the loaded area

q = uniformly distributed load

c = No. of concentric areas

s = No. of radial lines

Approximate method

The method discussed in the preceding sections are relatively more accurate, but are time consuming. Sometimes, the engineer is interested to estimate the vertical stresses approximately. For preliminary designs, thus saving time and labour without sacrificing accuracy to any significant degree.

They are also used to determine the stress distribution in soil under the influence of complex loading and/ or shapes of loaded areas.

Two commonly used approximate methods are:

Equivalent point- load method

The vertical stress at a point under a loaded area of any of any shape can be determined by dividing the loaded area into small area and replacing the distributed load on each on small area by an equivalent point load acting at the centroid of the small area. The principle of superposition is then applied and the required stress at a specified point is obtained by summing up the contributions of the individual. Point loads from each of the units by applying the approximate point load formula, such as that of Boussinesq‟s or Westergaard‟s.

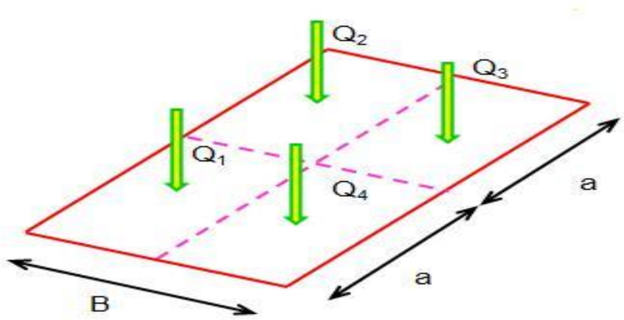


Fig.2.17 Equivalent point- load method

As shown in the Fig.2.17, if a square area of size B is acted on by a uniform load q, the same area can be divided into four small area. And the load on each area can be converted into an equivalent point load assumed to act at its centroid. Then the vertical stress at any point below or outside the loaded area is equal to the sum of the vertical stresses due to these equivalent point loads. Then

$$\sigma_Z = \frac{[Q_1(I_B)_1 + Q_2(I_B)_2 + Q_3(I_B)_3 + \ldots\ldots\ldots + Q_n(I_B)_n]}{Z^2}$$

$$\sigma_Z = \frac{1}{Z^2} \sum_{i=1}^{n} Q_i(I_B)_i$$

2.12 Contact pressure

The upward pressure due to soil on the underside of the footing or foundation is termed contact pressure.

In the derivations of vertical stress below the loaded areas using Boussinesq‟s theory or Westergaard‟s theory, it has been assumed that the footing is flexible and the contact pressure distribution is uniform and equal to „q‟. Actual footings are not flexible as assumed. The actual distribution of the contact pressure depends on a number of factors.

2.12.1 Factors affecting contact pressure distribution

The factors are:

1. Flexural rigidity of base of footing
2. Type of soil
3. Confinement

Flexural rigidity of base of footing

Uniform loading on a flexible base induces uniform contact pressure on any type of soil, while a rigid base induces non-uniform pressure. Foundation bases are usually thick massive concrete structures, which cannot be treated as ideally flexible.

Type of soil

The contact pressure distribution also depends on the elastic properties of the soil. The elastic properties of soil depends on the type of soil.

a. Sandy soil

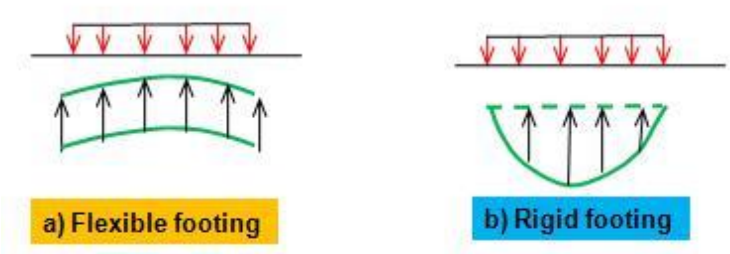


Fig. 2.19 Contact pressure diagram on sand

Fig 2.19 a & b shows the qualitative contact pressure distribution under flexible and rigid footing resting on a sandy soil and subjected to a uniformly distributed load q. when the footing is flexible, the edges undergo a large settlement than at centre. The soil at centre is confined and therefore has a high modulus of elasticity and deflects less for the same contact pressure. The contact pressure is uniform.

When the footing is rigid the settlement is uniform. The contact pressure is parabolic with zero intensity at the edge sand maximum at the centre.

b. Clayey soils

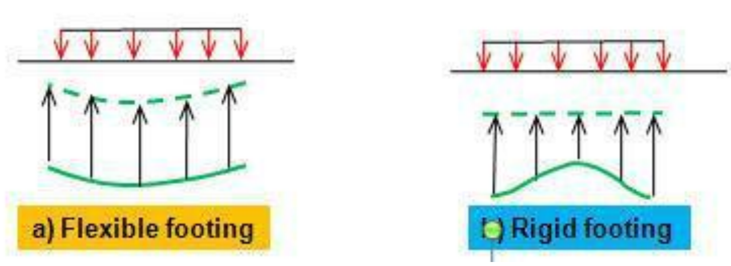


Fig. 2.20 Contact pressure diagram on saturated clay

Fig. 2.20 shows the qualitative contact pressure distribution under flexible and rigid footings resting on saturated clay and subjected to a uniformly distributed load q.

When the footing is flexible, it deforms into the shape of a bowel, with the maximum deflection at the centre. The contact pressure distribution is uniform

If the footing is rigid, the settlement is uniform. The contact pressure distribution is minimum at the centre and the maximum at the edges(infinite theoretically). The stresses at the edges in real soil cannot be infinite as theoretically determined for an elastic mass. In real soils, beyond a certain limiting values of stress, the plastic flow occurs and the pressure becomes infinite as shown in Fig.2.20

c. C-Ø soil

For a c – Ø soil, the contact pressure for a flexible footing will be uniform as shown in Fig. 3.21(a). For a rigid footing, the pressure distribution will be as shown in the Fig. 3.21 (b), it is more at the edge and less at the centre.

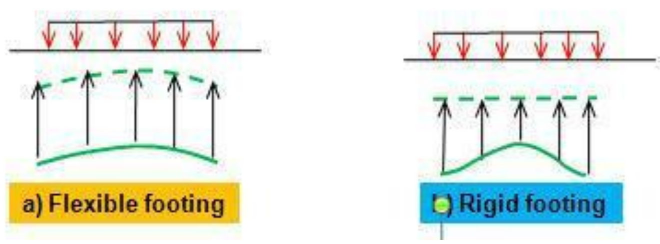


Fig. 2.21 Contact pressure diagram on c – Ø soil

Confinement

For surface loading in sand contact pressure is zero and for clayey soils, it is very high. When the footings are confined then the edge stresses and the contact pressure distribution changes. In sand, if the foundation is embedded or confined, then there would be some finite contact pressure at the edges. In clayey soil the contact pressure at the edges slightly reduces as confinement increase at the edges to surface loading

The more the foundation is below the surface of the sand, the more the shear resistance developed at the edges due to increase in the overburden pressure and as a consequence, the contact pressure distribution tends to be more uniform as compared to being parabolic to surface loading.

2.13 Assignment Questions

1. List the assumption of Boussinesq analysis for the pressure distribution in a soil layer.
2. What do you understand by “pressure bulb”? Illustrate with sketches.
3. Using Boussinesq‟s equation, construct isobar of intensity 0.1 Q, where Q is point load acting on surface.
4. Derive an expression to find vertical pressure under a uniformly loaded circular area

2.14 OUTCOMES

- Various methods to determine stresses in soils
- Understanding of contact pressure
- Knowledge of estimation of settlement in soils

2.15 Further reading

- http://nptel.ac.in/courses/105103097/20
- http://environment.uwe.ac.uk/geocal/SoilMech/stresses/stresses.htm

Module-3
LATERAL EARTH PRESSURE

Structure

3.0 Objectives

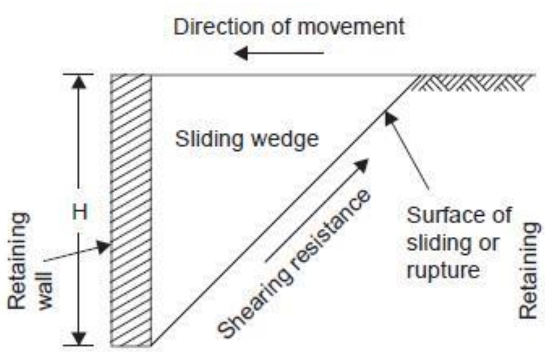
- To understand earth pressure and its importance
- To determine magnitude of LEP by applying Rankines and Coulomb's theory and graphical methods
- To understand stability slopes, its causes and importance
- To estimate factor of safety of slopes by analytical and graphical method

3.1 Introduction

Soil is neither a solid nor a liquid, but it exhibits some of the characteristics of both. One of the characteristics similar to that of a liquid is its tendency to exert a lateral pressure against any object in contact. This important property influences the design of retaining walls, abutments, bulkheads, sheet pile walls, basement walls and underground conduits which retain or support soil, and, as such, is of very great significance. Retaining walls are constructed in various fields of civil engineering, such as hydraulics and irrigation structures, highways, railways, tunnels, mining and military engineering.

3.2 Lateral earth pressures

Lateral earth pressure is the force exerted by the soil mass upon an earth- retaining structure, such as a retaining wall. There are two distinct kinds of lateral earth pressure; the nature of each is to be clearly understood. First, let us consider a retaining wall which holds back a mass of soil. The soil exerts a push against the wall by virtue of its tendency to slip laterally and seek its natural slope or angle of repose, thus making the wall to move slightly away from the backfilled soil mass. This kind of pressure is known as the _active‘ earth pressure of the soil. The soil, being the actuating element, is considered to be active and hence the name active earth pressure. Next, let us imagine that in some manner the retaining wall is caused to move toward the soil. In such a case the retaining wall or the earth-retaining structure is the actuating element and the soil provides the resistance which soil develops in response to movement of the structure toward it is called the _passive earth pressure‘, or more appropriately _passive earth resistance‘ which may be very much greater than the active earth pressure. The surface over which the sheared-off soil wedge tends to slide is referred to as the surface of _sliding‘ or _rupture‘. The limiting values of both the active earth pressure and passive earth resistance for a given soil depend upon the amount of movement of the structure. In the case of active pressure, the structure tends to move away from the soil, causing strains in the soil mass, which in turn, mobilise shearing stresses; these stresses help to support the soil mass and thus tend to reduce the pressure exerted by the soil against the structure. This is indicated in Fig.



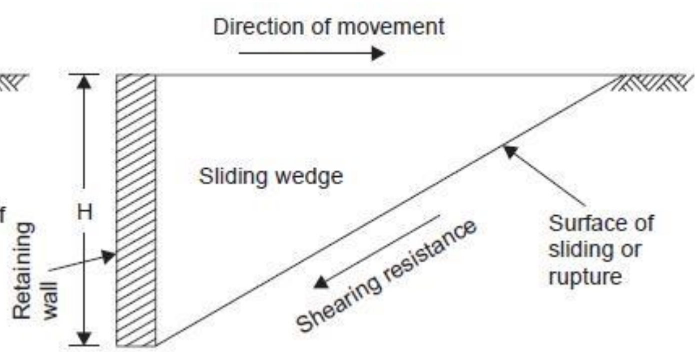


Fig.Conditions in the case of pressure passive earth resistance Fig.Conditions in the case of active

In the case of passive earth resistance also, internal shearing stresses develop, but act in the opposite direction to those in the active case and must be overcome by the movement of the structure. This difference in direction of internal stresses accounts for the difference in magnitude between the active earth pressure and the passive earth resistance. The conditions obtaining in the passive case are indicated in Fig.

3.3 Earth pressure at rest

Active pressures are accompanied by movements directed away from the soil, and passive resistances are accompanied by movements towards the soil. Logically, therefore, there must be a situation intermediate between the two when the retaining structure is perfectly stationary and does not move in either direction. The pressure which develops in this condition is called _earth pressure at rest‘. Its value is a little larger than the limiting value of active pressure, but is considerably less than the maximum passive resistance. This is indicated in Fig.

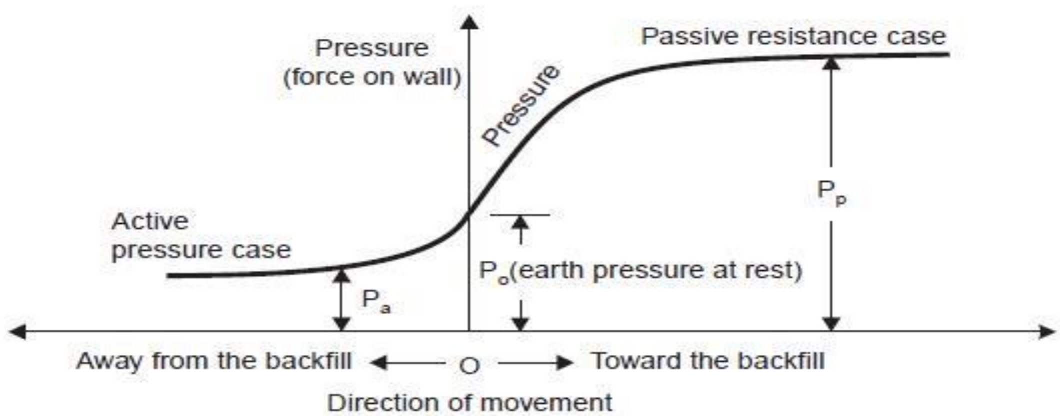


Fig. Relation between lateral earth pressure and movement of wall

Earth pressure at rest may be obtained theoretically from the theory of elasticity applied to an element of soil, remembering that the lateral strain of the element is zero. Referring to Fig. (*a*), the principal stresses acting on an element of soil situated at a depth *z* from the surface in semi-infinite, elastic, homogeneous and isotropic soil mass are σ_v, σ_h and σ_h as shown. σ_v and σ_h denoting the stresses in the vertical and horizontal directions respectively.

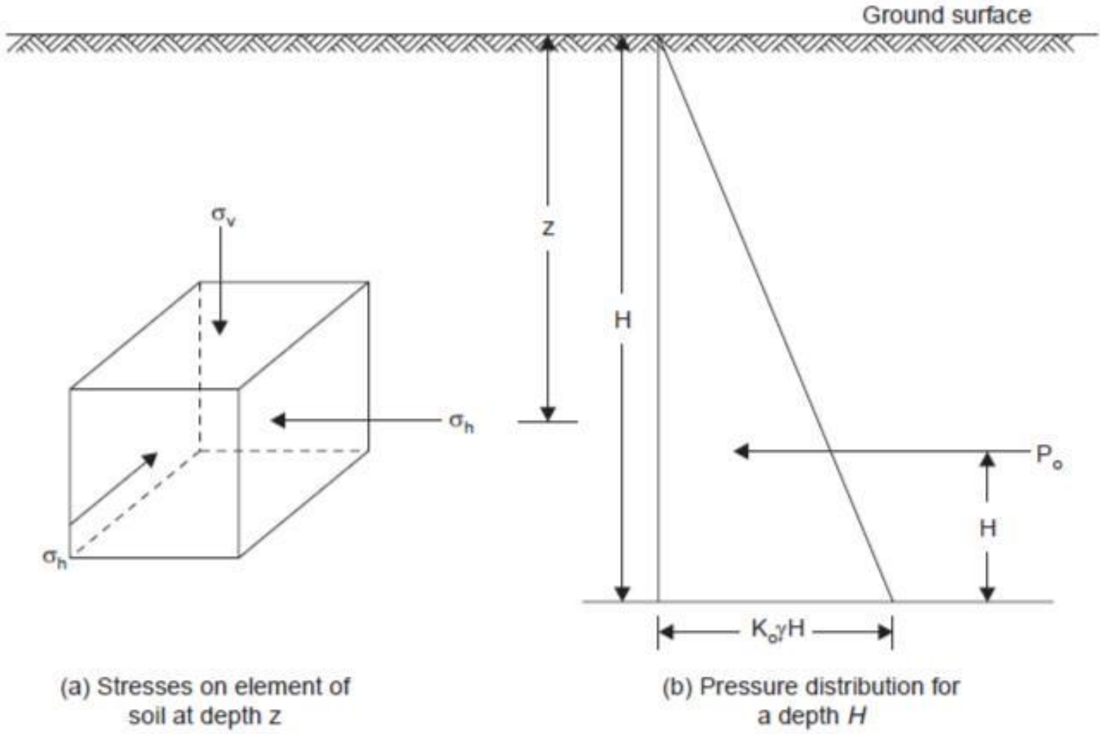


Fig. Stress conditions relating to earth pressure at rest

The soil deforms vertically under its self-weight but is prevented from deforming laterally because of an infinite extent in all lateral directions. Let *Es* and ν be the modulus of elasticity and Poisson's ratio of the soil respectively.

$$\text{Lateral strain,} \qquad \varepsilon_h = \frac{\sigma_h}{E_s} - \upsilon\left(\frac{\sigma_v}{E_s} + \frac{\sigma_h}{E_s}\right) = 0$$

$$\therefore \qquad \frac{\sigma_h}{\sigma_v} = \frac{\upsilon}{(1-\upsilon)}$$

But $\sigma v = \gamma . z$, where γ is the appropriate unit weight of the soil depending upon its condition.

$$\sigma_h = \left(\frac{\upsilon}{1-\upsilon}\right) . \gamma . z$$

$$K_0 = \left(\frac{\upsilon}{1-\upsilon}\right)$$

$$\sigma_h = K_0 .\ \gamma.z$$

The distribution of the earth pressure at rest with depth is obviously linear (or of hydrostatic nature) for constant soil properties such as *E*, υ, and γ, as shown in Fig. If a structure such as a retaining wall of height *H* is interposed from the surface and imagined to be held without yield, the total thrust on the wall unit length *P*0, is given by

$$P_0 = \int_0^H \sigma_h .dz = \int_0^H K_0 .\gamma z.dz = \frac{1}{2} K_0 .\gamma .H^2$$

This is considered to act at (1/3) *H* above the base of wall. As has been indicated in the previous chapter, choosing an appropriate value for the Poisson's ratio, ν, is by no means easy; this is the limitation in arriving at K_0 from equation. Various researchers proposed empirical relationships for K_0, some of which are given below.

$$K_0 = (1 - \sin\phi') \text{ (Jaky, 1944)}$$
$$K_0 = 0.9\ (1 - \sin\phi') \text{ (Fraser, 1957)}$$
$$K_0 = 0.19 + 0.233 \log I_p \text{ (Kenney, 1959)}$$
$$K_0 = [1 + (2/3) \sin\phi'] \left(\frac{1-\sin\phi}{1+\sin\phi}\right) \text{ (Kezdi, 1962)}$$
$$K_0 = (0.95 - \sin\phi') \text{ (Brooker and Ireland, 1965)}$$

φ′ in these equations represents the effective angle of friction of the soil and *Ip*, the plasticity index. Brooker and Ireland (1965) recommend Jaky's equation for cohesioness soils and their own equation, given above, for cohesive soils. However, Alpan (1967) recommends Jaky's equation for cohesionless soils and Kenney equation for cohesive soils as does Kenney (1959).

Certain values of the coefficient of earth pressure at rest are suggested for different soils, based on field data, experimental evidence and experience. These are given in Table

S.No.	Soil			K_0
1	Loose Sand (e = 0.8)			
		dry	...	0.64
		Saturated	...	0.46
2	Dense sand (e = 0.6)			
		dry	...	0.49
		saturated	...	0.36
3	Sand (compacted in layers)		...	0.80
4	Soft clay (I_p = 30)		...	0.60
5	Hard clay (I_p = 9)		...	0.42
6	Undisturbed Silty clay (I_p = 45)		...	0.57

3.4 Earth pressure theories

The magnitude of the lateral earth pressure is evaluated by the application of one or the other of the so-called _lateral earth pressure theories' or simply _earth pressure theories'. The problem of determining the lateral pressure against retaining walls is one of the oldest in the field of engineering. A French military engineer, Vauban, set forth certain rules for the design of revetments in 1687. Since then, several investigators have proposed many theories of earth pressure after a lot of experimental and theoretical work. Of all these theories, those given by Coulomb and Rankine stood the test of time and are usually referred to as the ―Classical earth pressure theories‖. These theories are considered reliable in spite of some limitations and are considered basic to the problem. These theories have been developed originally to apply to cohesionless soil backfill, since this situation is considered to be more frequent in practice and since the designer will be on the safe side by neglecting cohesion. Later researchers gave necessary modifications to take into account cohesion, surcharge, submergence, and so on. Some have evolved graphical procedures to evaluate the total thrust on the retaining structure. Although Coulomb presented his theory nearly a century earlier to Rankine's theory, Rankine's theory will be presented first due to its relative simplicity.

3.4.1 Rankine's theory

Rankine (1857) developed his theory of lateral earth pressure when the backfill consists of dry, cohesionless soil. The theory was later extended by Resal (1910) and Bell (1915) to be applicable to cohesive soils.

The following are the important assumptions in Rankine's theory:

(*i*) The soil mass is semi infinite, homogeneous, dry and cohesionless.

(*ii*) The ground surface is a plane which may be horizontal or inclined.

(*iii*) The face of the wall in contact with the backfill is vertical and smooth. In other words, the friction between the wall and the backfill is neglected (This amounts to ignoring the presence of the wall).

(*iv*) The wall yields about the base sufficiently for the active pressure conditions to develop; if it is the passive case that is under consideration, the wall is taken to be pushed sufficiently towards the fill for the passive resistance to be fully mobilised. (Alternatively, it is taken that the soil mass is stretched or gets compressed adequately for attaining these states, respectively. Friction between the wall and fill is supposed to reduce the active earth pressure on the wall and increase the passive resistance of the soil. Similar is the effect of cohesion of the fill soil). Thus it is seen that, by neglecting wall friction as also cohesion of the backfill, the geotechnical engineer errs on the safe side in the computation of both the active pressure and passive resistance. Also, the fill is usually of cohesionless soil, wherever possible, from the point of view of providing proper drainage.

3.4.2 Coulomb's wedge theory

The primary assumptions in Coulomb's wedge theory are as follows:

1. The backfill soil is considered to be dry, homogeneous and isotropic; it is elastically underformable but breakable, granular material, possessing internal friction but no cohesion.

2. The rupture surface is assumed to be a plane for the sake of convenience in analysis. It passes through the heel of the wall. It is not actually a plane, but is curved and this is known to Coulomb.

3. The sliding wedge acts as a rigid body and the value of the earth thrust is obtained by considering its equilibrium.

4. The position and direction of the earth thrust are assumed to be known. The thrust acts on the back of the wall at a point one-third of the height of the wall above the base of the wall and makes an angle δ, with the normal to the back face of the wall. This is an angle of friction between the wall and backfill soil and is usually called _wall friction‘.

5. The problem of determining the earth thrust is solved, on the basis of two-dimensional case of _plane strain‘. This is to say that, the retaining wall is assumed to be of great length and all conditions of the wall and fill remain constant along the length of the wall. Thus, a unit length of the wall perpendicular to the plane of the paper is considered.

6. When the soil wedge is at incipient failure or the sliding of the wedge is impending, the theory gives two limiting values of earth pressure, the least and the greatest (active and passive), compatible with equilibrium. The additional inherent assumptions relevant to the theory are as follows:

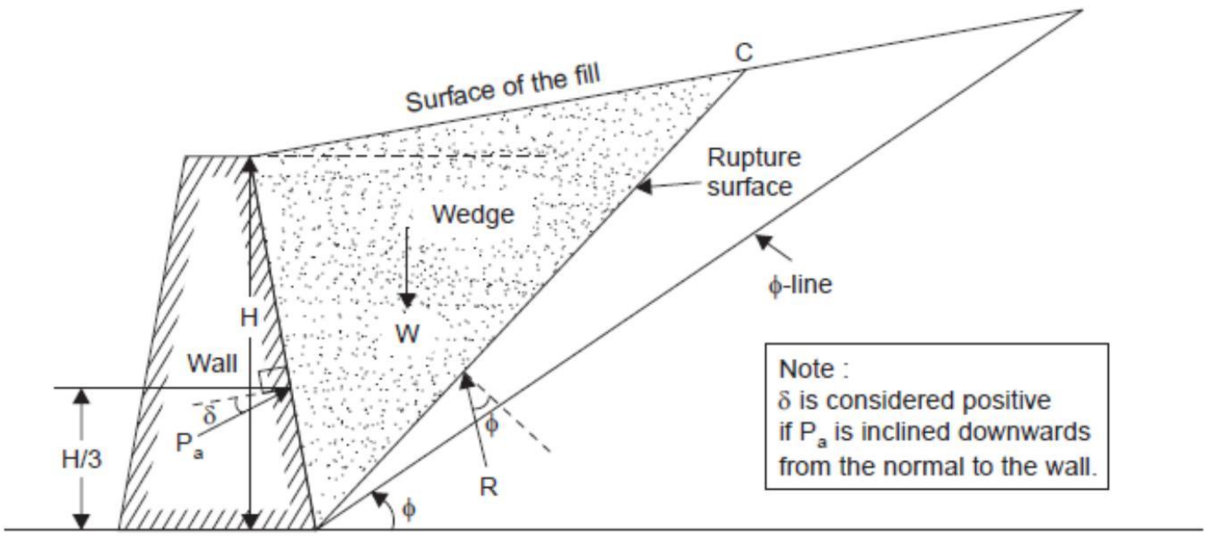
7. The soil forms a natural slope angle, φ, with the horizontal, without rupture and sliding. This is called the angle of repose and in the case of dry cohesionless soil, it is nothing but the angle of internal friction. The concept of friction was understood by Coulomb.

8. If the wall yields and the rupture of the backfill soil takes place, a soil wedge is torn off from the rest of the soil mass. In the active case, the soil wedge slides sideways and downward over the rupture surface, thus exerting a lateral pressure on the wall. In the case of passive earth resistance, the soil wedge slides sideways and upward on the rupture surface due to the forcing of the wall against the fill. These are illustrated in Fig.

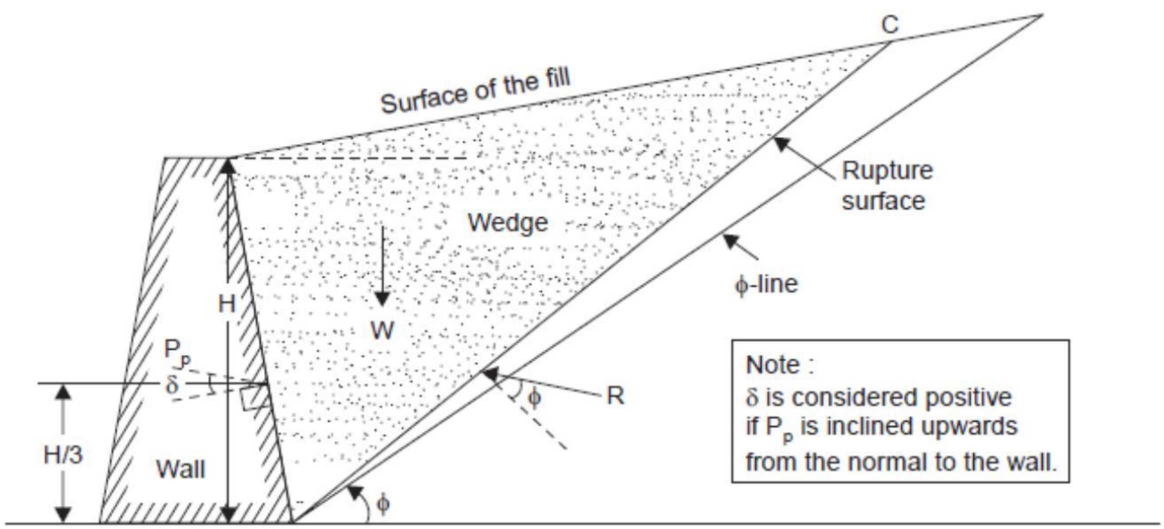
9. For a rupture plane within the soil mass, as well as between the back of the wall and the soil, Newton‘s law of friction is valid (that is to say, the shear force developed due to friction is the coefficient of friction times the normal force acting on the plane). This angle of friction, whose tangent is the coefficient of friction, is dependent upon the physical properties of the materials involved.

10. The friction is distributed uniformly on the rupture surface.

11. The back face of the wall is a plane.



(a) Active earth pressure



(b) Passive earth resistance

Limitations of coulomb's theory

Also note that Coulomb‘s theory treats the soil mass in the sliding wedge in its entirety. The assumptions permit one to treat the problem as a statically determinate one. Coulomb‘s theory is applicable to inclined wall faces, to a wall with a broken face, to a sloping backfill curved backfill surface, broken backfill surface and to concentrated or distributed surcharge loads. One of the main deficiencies in Coulomb‘s theory is that, in general, it does not satisfy the static equilibrium condition occurring in nature. The three forces (weight of the sliding wedge, earth pressure and soil reaction on the rupture surface) acting on the sliding wedge do not meet at a common point, when the sliding surface is assumed to be planar. Even the wall friction was not originally considered but was introduced only some time later. Regardless of this deficiency and other assumptions, the theory gives useful results in practice; however, the soil constants should be determined as accurately as possible.

3.5 Lateral Earth Pressure of Cohesive Soil

3.5.1 Active Earth Pressure of Cohesive Soil

The lateral earth pressure of cohesive soil may be obtained from the Coulomb‘s wedge theory; however, one should take cognisance of the tension zone near the surface of the cohesive backfill and consequent loss of contact and loss of adhesion and friction at the back of the wall and along the plane of rupture, so as to avoid getting erroneous results. The trial wedge method may be applied to this case as illustrated in Fig. The following five forces act on a trial wedge:

1. Weight of the wedge including the tension zone, *W*.
2. Cohesion along the wall face or adhesion between the wall and the fill, *Ca*.
3. Cohesion along the rupture plane, *C*.
4. Reaction on the plane of failure, *R*, acting at φ to the normal to the plane of failure.
5. Active thrust, *Pa*, acting at δ to the normal to the face of the wall.
6. The total adhesion force, *Ca*, is given by

$$C_a = c_a \cdot \overrightarrow{BF}$$

where c_a is the unit adhesion between the wall and the fill, which cannot be greater than the unit cohesion, c, of the soil. c_a may be obtained from tests; however, in the absence of data, c_a may be taken as equal to c for soils with c up to 50 kN/m^2, *ca* may be limited to 50 kN/m^2 for soils with c greater than this value. (Smith, 1974).

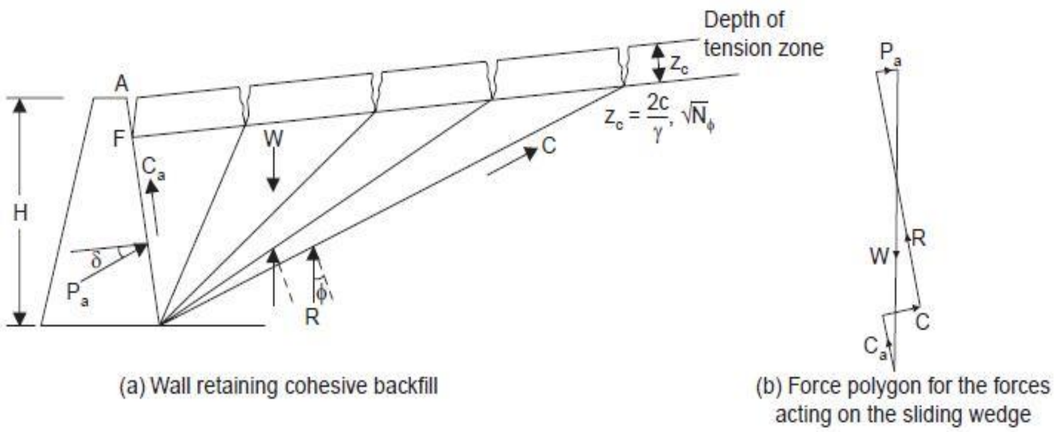


Fig. Active earth pressure of cohesive soil—trial wedge method—Coulomb‘s theory

The total cohesion force, *C*, is given by

$$C = c.\overrightarrow{BC}$$

c being the unit cohesion of the fill soil and BC→ is the length of the rupture plane. The three forces W, Ca, and C are fully known and the directions of the other two unknown forces R and Pa are known; the vector polygon may therefore be completed as shown in Fig. (b), and the value of Pa may be scaled-off. A number of such trial wedges may be analyzed and the maximum of all Pa values chosen as the active thrust. The rupture plane may also be located. The final value of the thrust on the wall is the resultant of Pa and Ca. Culmann‘s method may also be adapted to suit this case, as illustrated in fig.

3.5.2 Passive Earth Pressure of Cohesive Soil

The procedure adopted to determine the active earth pressure of cohesive soil from Coulomb‘s theory may also be used to determine the passive earth resistance of cohesive soil. The points of difference are that the signs of friction angles, φ and δ, will be reversed and the directions of C_a and C also get reversed. Either the trial wedge approach or Culmann‘s approach may be used but one has also to consider the effect of the tensile zone in reducing C_a and C. However, it must be noted that the Coulomb theory with plane rupture surfaces is not applicable to the case of passive resistance. Analysis must be carried out, strictly speaking, using curved rupture surfaces such as logarithmic spirals (Terzaghi, 1943), so as to avoid overestimation of passive resistance.

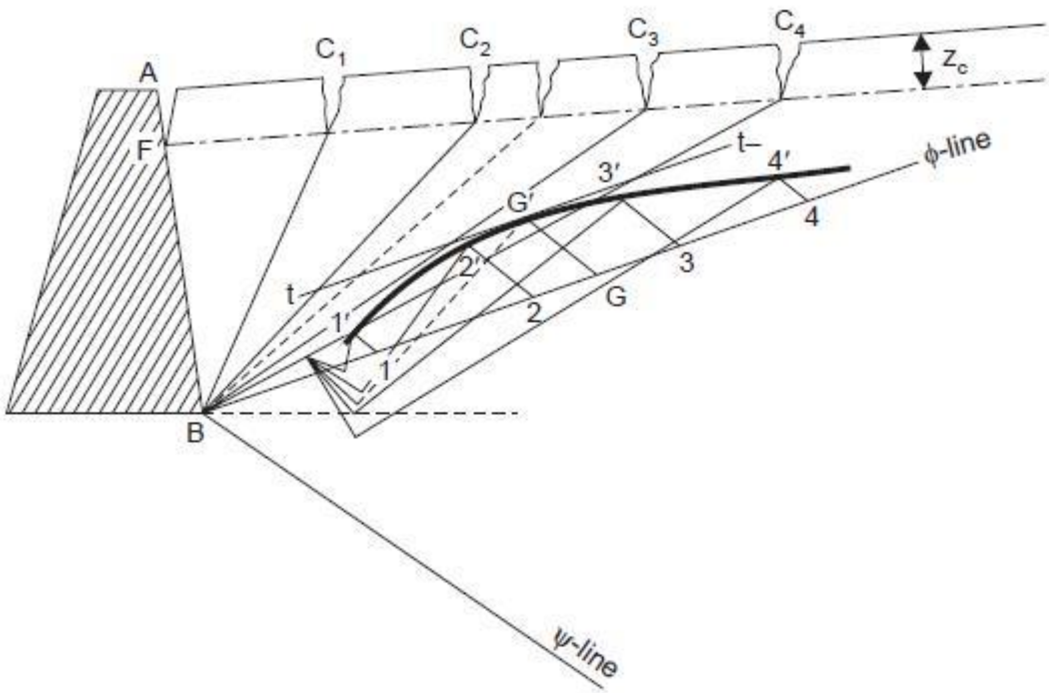


Fig. Culmann‘s method adapted to allow for cohesion

3.6 Lateral earth pressure for cohesionless soil

3.6.1 Active Earth Pressure of Cohesionless Soil

A simple case of active earth pressure on an inclined wall face with a uniformly sloping backfill may be considered first. The backfill consists of homogeneous, elastic and isotropic cohesionelss soil. A unit length of the wall perpendicular to the plane of the paper is considered. The forces acting on the sliding wedge are (*i*) *W*, weight of the soil contained in the sliding wedge, (*ii*) *R*, the soil reaction across the plane of sliding, and (*iii*) the active thrust P_a against the wall, in this case, the reaction from the wall on to the sliding wedge, as shown in Fig.

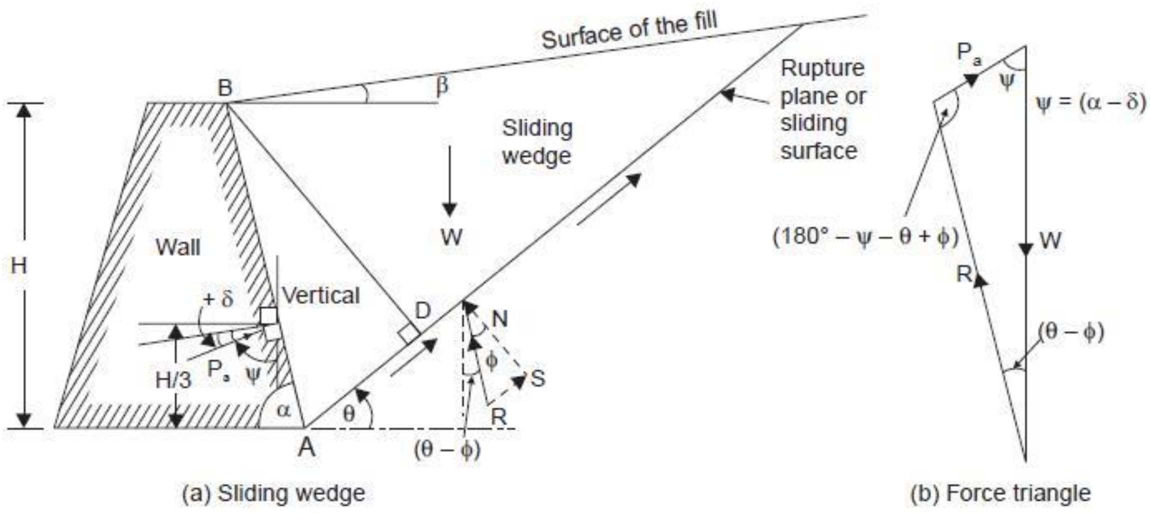


Fig. Active earth pressure of cohesionless soil—Coulomb‘s theory

The triangle of forces is shown in Fig. (*b*). With the nomenclature of Fig. one may proceed as follows for the determination of the active thrust, *Pa*: *W* = γ (area of wedge *ABC*)

$$\Delta ABC = \frac{1}{2} AC.\ BD,\ BD \text{ being the altitude on to } AC.$$

$$AC = AB.\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}$$

$$BD = AB.\ \sin\ (\alpha+\theta)$$

$$AB = \frac{H}{\sin\alpha}$$

Substituting and simplifying,

$$W = \gamma\frac{H^2}{2\sin^2\alpha}.\sin(\theta+\alpha).\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}$$

From the triangles of forces,

$$\frac{P_a}{\sin(\theta-\phi)} = \frac{W}{\sin(180^\circ-\psi-\theta+\phi)}$$

$$P_a = W.\frac{\sin(\theta-\phi)}{\sin(180^\circ-\psi-\theta+\phi)}$$

Substituting for *W*,

$$P_a = \frac{1}{2}\frac{\gamma H^2}{\sin^2\alpha}.\frac{\sin(\theta-\phi)}{\sin(180^\circ-\psi-\theta+\phi)}.\frac{\sin(\theta+\alpha).\sin(\alpha+\beta)}{\sin(\theta-\beta)}$$

The maximum value of *Pa* is obtained by equating the first derivative of *Pa* with respect to θ to zero;

$$\text{or } \frac{\partial P_a}{\partial\theta} = 0, \text{ and substituting the corresponding value of } \theta.$$

The value of *Pa* so obtained is written as

$$P_a = \frac{1}{2}\gamma.H^2.\frac{\sin^2(\alpha+\phi)}{\sin^2\alpha\sin(\alpha-\delta)\left[1+\sqrt{\frac{\sin(\phi+\delta)\sin(\phi-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2}$$

This is usually written as

$$P_a = \frac{1}{2}\gamma H^2.K_a,$$

$$K_a = \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha.\sin(\alpha-\delta)\left[1+\sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2}$$

K_a being the coefficient of active earth pressure. For a vertical wall retaining a horizontal backfill for which the angle of wall friction is equal to φ, *Ka* reduces to

$$K_a = \frac{\cos\phi}{(1+\sqrt{2}\sin\phi)^2}$$

by substituting α = 90°, β = 0°, and δ = φ. For a smooth vertical wall retaining a backfill with horizontal surface, α = 90°, δ = 0, and β = 0;

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2(45° - \phi/2) = 1/N_\phi,$$

which is the same as the Rankine value. In fact, for this simple case, one may proceed from fundamentals as follows:

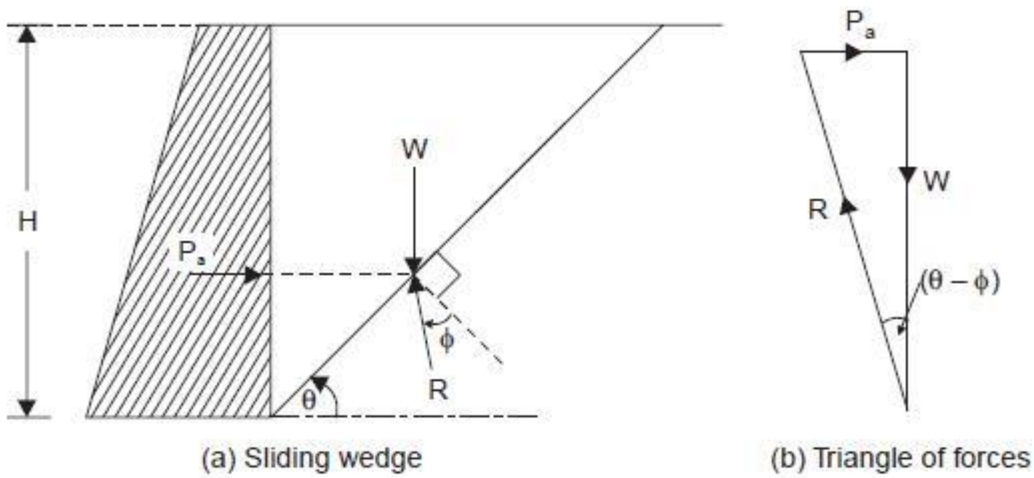


Fig. Active earth pressure of cohesionless soil special case: α = 90°, δ = β = 0°

$$P_a = W\tan(\theta - \phi),$$
$$W = \frac{1}{2}\gamma H^2.\cot\phi$$
$$P_a = \frac{1}{2}\gamma H^2\cot\theta\tan(\theta - \phi)$$

For maximum value of P_a, $\frac{\partial P_a}{\partial\theta} = 0$

$$\therefore \qquad \frac{\partial P_a}{\partial\theta} = \frac{1}{2}\gamma H^2\left[-\frac{\tan(\theta-\phi)}{\sin^2\theta} + \frac{\cot\theta}{\cos^2(\theta-\phi)}\right] = 0$$

or $$\frac{-\sin(\theta-\phi)\cos(\theta-\phi) + \sin\theta\cos\theta}{\sin^2\theta\cos^2(\theta-\phi)} = 0$$

or $\sin\phi\cos(2\theta - \phi) = 0$, on simplification.

$\therefore \qquad \cos(2\theta - \phi) = 0 \qquad$ or $\qquad \theta = 45° + \phi/2$

$$P_a = \frac{1}{2}\gamma H^2\tan^2(45° - \phi/2)$$

as obtained by substitution in the general equation. Ironically, this approach is sometimes known as _Rankine's method of Trial Wedges'. A few representative values of *Ka* from Eq. 13.34 for certain values of φ, δ, α and β are

δ↓φ→	20°	30°	40°
		α = 90°, β = 0°	
0°	0.49	0.33	0.22
10°	0.45	0.32	0.21
20°	0.43	0.31	0.20
30°	...	0.30	0.20
		α = 90°, β = 10°	
0°	0.51	0.37	024
10°	0.52	0.35	0.23
20°	0.52	0.34	0.22
30°	...	0.33	0.22
		α = 90°, β = 20°	
0°	0.88	0.44	0.27
10°	0.90	0.43	0.26
20°	0.94	0.42	0.25
30°	...	0.42	0.25

It may be observed that the theoretical solution is thus rather complicated even for relatively simple cases. This fact has led to the development of graphical procedures for arriving at the total thrust on the wall. Poncelet (1840), Culmann (1866), Rebhann (1871), and Engessor (1880) have given efficient graphical solutions, some of which will be dealt with in the subsequent subsections. An obvious grpahical approach that suggests itself if the ―Trial-Wedge method‖. In this method, a few trial rupture surfaces are assumed at varying inclinations, θ, with the horizontal and passing through the heel of the wall; for each trial surface the triangle of forces is completed and the value of *Pa* found. A θ – *Pa* plot is made which should appear somewhat as shown in Fig. If an adequate number of intelligently planned trial rupture surface are analysed. The maximum value of *Pa* from this plot gives the anticipated total active thrust on the wall per lineal unit and the corresponding value of θ, the inclination of the most probable rupture surface. Wall friction At this juncture, a few comments on wall friction may be appropriate. In the active case, the outward stretching leads to a downward motion of the backfill soil relative to the wall. Such a downward shear force upon the wall is called _positive' wall friction for the active case.

This leads to the upward inclination of the active thrust exerted on the sliding wedge as shown in Fig. (*a*). This means that the active thrust exerted on the wall will be directed with a downward inclination. In the passive case, the horizontal compression must be accompanied by an upward bulging of the soil and hence there tends to occur an upward shear on the wall. Such an upward shear on the wall is said to be ‗positive' wall friction for the passive case. This leads to the downward inclination of the passive thrust exerted on the sliding wedge as shown in fig. 13.24 (*a*); this means that the passive resistance exerted on the wall will be directed with upward inclination. In the active case wall friction is almost always positive. Sometimes, under special conditions, such as when part of the backfill soil immediately behind the wall is excavated for repair purposes and the wall is braced against the remaining earth mass of the backfill, negative wall friction might develop. Either positive or negative wall friction may develop in the passive case. This sign of wall friction must be determined from a study of motions expected for each field situation. Once wall friction is present, the shape of the rupture surface is curved and not plane. The nature of the surface for positive and negative values of wall friction is shown in Figs. (*a*) and (*b*), respectively.

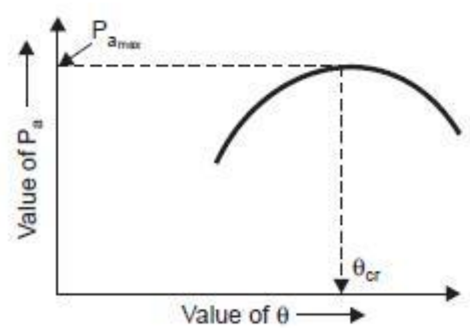


Fig. Angle of inclination of trial rupture plane versus active thrust

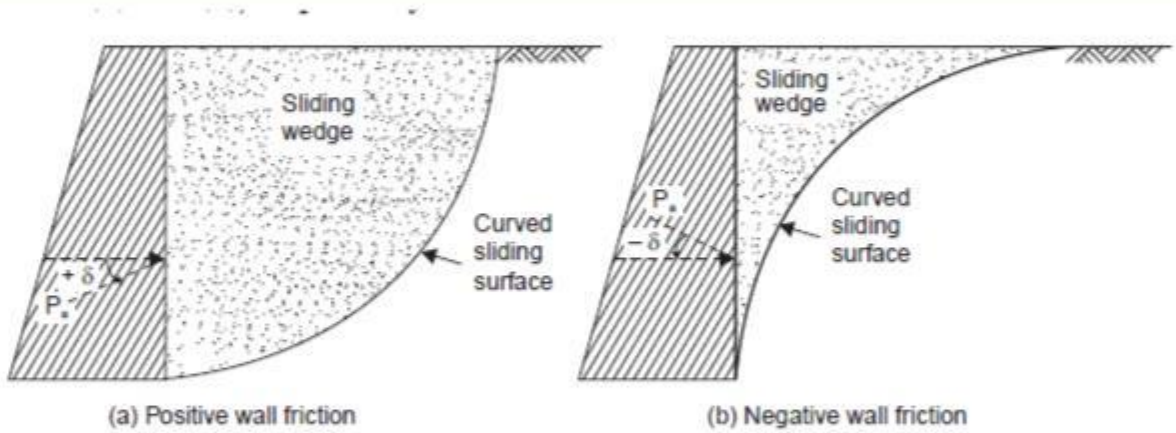


Fig. Positive and negative wall friction for active case along with probable shape of sliding surface

The angle of wall friction, δ, will not be greater than φ; at the maximum it can equal φ, for a rough wall with a loose fill. For a wall with dense fill, δ will be less than φ. It may range from 12 φ to 34 φ in most cases; it is usually assumed as (2/3) φ in the absence of precise data. The possibility of δ shifting from +φ to –φ in the worst case should be considered in the design of a retaining wall. The value of Ka for the case of a vertical wall retaining a fill with a level surface, in which φ ranges from 20° to 40° and δ ranges from 0° to φ, may be obtained from the chart given in Fig.

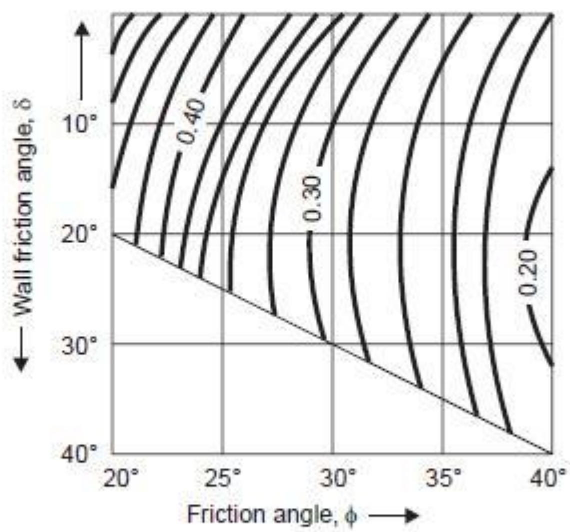


Fig. Coefficient of active pressure as a function of wall friction

The influence of wall friction on *Ka* may be understood from this chart to some extent. The assumption of plane failure in the active case of the Coulomb theory is in error by only a relatively small amount. It has been shown by Fellenius that the assumption of circular arcs for failure surfaces leads to active thrusts that generally do not exceed the corresponding values from the Coulomb theory by more than 5 per cent.

3.6.2 Passive Earth Pressure of Cohesionless Soil

The passive case differs from the active case in that the obliquity angles at the wall and on the failure plane are of opposite sign. Plane failure surface is assumed for the passive case also in the Coulomb theory but the critical plane is that for which the passive thrust is minimum. The failure plane is at a much smaller angle to the horizontal than in the active case, as shown in Fig. The triangle of forces with the usual nomenclature, the passive resistance *PP* may be determined as follows:

$$W = \frac{1}{2}\frac{\gamma H^2}{\sin^2\alpha}.\sin(\theta+\alpha).\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)},\ \text{as in the active case.}$$

From the triangle of forces

$$\frac{P_p}{\sin(\theta+\phi)} = \frac{W}{\sin(180° - \psi - \theta - \phi)}$$

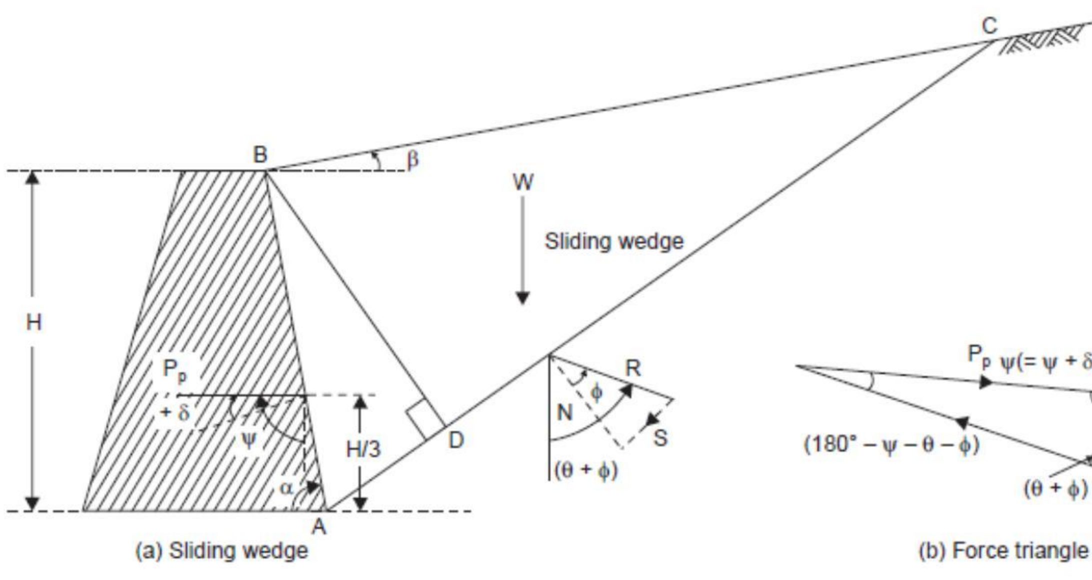


Fig. Passive earth pressure of cohesionless soil—Coulomb‘s theory

$$P_p = W.\frac{\sin(\theta+\phi)}{\sin(180^\circ-\psi-\theta-\phi)}$$

Substituting for *W,*

$$P_p = \frac{1}{2}.\frac{\gamma H^2}{\sin^2\alpha}.\sin(\theta+\alpha).\frac{\sin(\alpha+\beta)}{\sin(\theta-\beta)}.\frac{\sin(\theta+\alpha)}{\sin(180^\circ-\psi-\theta-\phi)}$$

The minimum value of *Pp* is obtained by differentiating with respect to θ

$$P_p = \frac{1}{2}\gamma H^2.K_p$$

$$K_p = \frac{\sin^2(\alpha-\phi)}{\sin^2\alpha.\sin(\alpha+\delta)\left[1-\sqrt{\frac{\sin(\theta+\delta).\sin(\phi+\beta)}{\sin(\alpha+\delta).\sin(\alpha+\beta)}}\right]^2}$$

KP being the coefficient of passive earth resistance. For a vertical wall retaining a horizontal backfill and for which the friction is equal to φ, α = 90°, β = 0°, and δ = φ, and *Kp* reduces to

$$K_p = \frac{\cos^2\phi}{\cos\phi\left[1-\sqrt{\frac{2\sin\phi\cos\phi.\sin\phi}{\cos\phi}}\right]^2}$$

$$K_p = \frac{\cos\phi}{(1-\sqrt{2}\sin\phi)^2}$$

For a smooth vertical wall retaining a horizontal backfill, α = 90°, β = 0° and δ = 0°;

$$K_p = \frac{\cos^2\phi}{(1-\sin\phi)^2} = \frac{1-\sin^2\phi}{(1-\sin\phi)^2} = \frac{(1+\sin\phi)}{(1-\sin\phi)} = \tan^2(45^\circ+\phi/2) = N_\phi,$$

which is the same as the Rankine value. For this simple case, it is possible to proceed from fundamentals, as has been shown for the active case. [(θ + φ) takes the place of (θ – φ) and (45° + φ/2) that of (45° – φ/2) in the work relating to the active case.] Coulomb‘s theory with plane surface of failure is valid only if the wall friction is zero in respect of passive resistance. The passive resistance obtained by plane failure surfaces is very much more than that obtained by assuming curved failure surfaces, which are nearer truth especially when wall friction is present. The error increases with increasing wall friction. This leads to errors on the unsafe side.

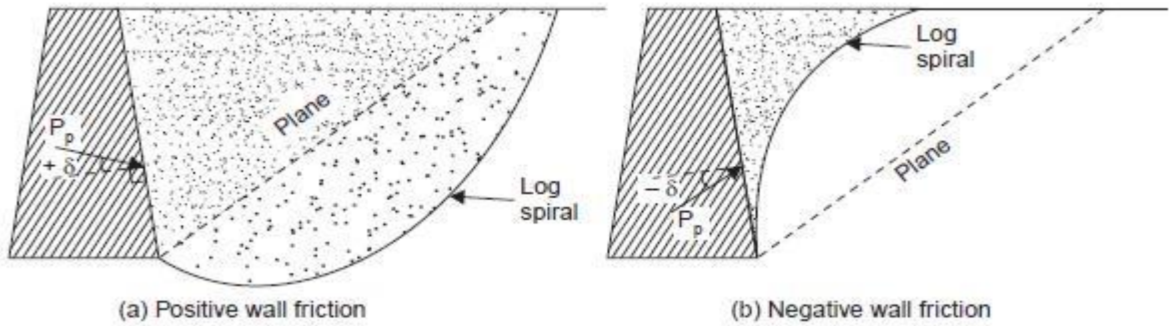


Fig. Curved failure surface for estimating passive resistance

Terzaghi (1943) has presented a more rigorous type of analysis assuming curved failure surface (logarithmic spiral form) which resembles those shown in Fig.. Terzaghi states that when δ is less than (1/3) φ, the error introduced by assuming plane rupture surfaces instead of curved ones in estimating the passive resistance is not significant; when δ is greater than (1/3) φ, the error is significant and hence cannot be ignored. This situation calls for the use of analysis based on curved rupture surfaces as given by Terzaghi; alternatively, charts and tables prepared by Caquot and Kerisel (1949) may be used. Extracts of such results are presented in Table 13.3 and Fig.

δ ↓ ϕ →	*10°*	*20°*	*30°*	*40°*
0°	1.42	2.04	3.00	4.60
ϕ/2	1.56	2.60	4.80	10.40
ϕ	1.65	3.00	6.40	17.50
−ϕ	0.72	0.58	0.54	0.52

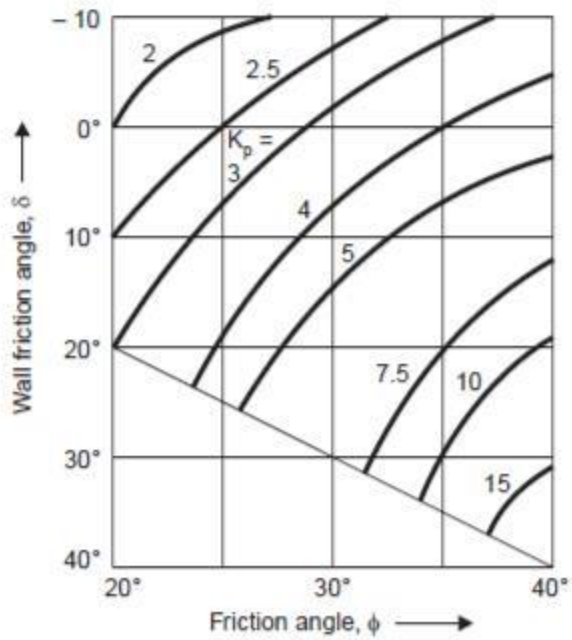


Fig. Chart for passive pressure coefficient(After Caquot and Kerisel, 1949)

Alternatively, Sokolovski‘s (1965) method may be used. This also gives essentially the same results. The theoretical predictions regarding passive resistance with wall friction are not well confirmed by experimental evidence as those regarding active thrust and hence cannot be used with as much confidence. Tschebotarioff (1951) gives the results of a few large-scale laboratory tests in this regard.

3.7 Rebhann's Condition and Graphical Method

Rebhann (1871) is credited with having presented the criterion for the direct location of the failure plane assumed in the Coulomb‘s theory. His presentation is somewhat as follows: Figure (*a*) represent a retaining wall retaining a cohesionless backfill inclined at +β to the horizontal. Let *BC* be the failure plane, the position of which is to be determined.

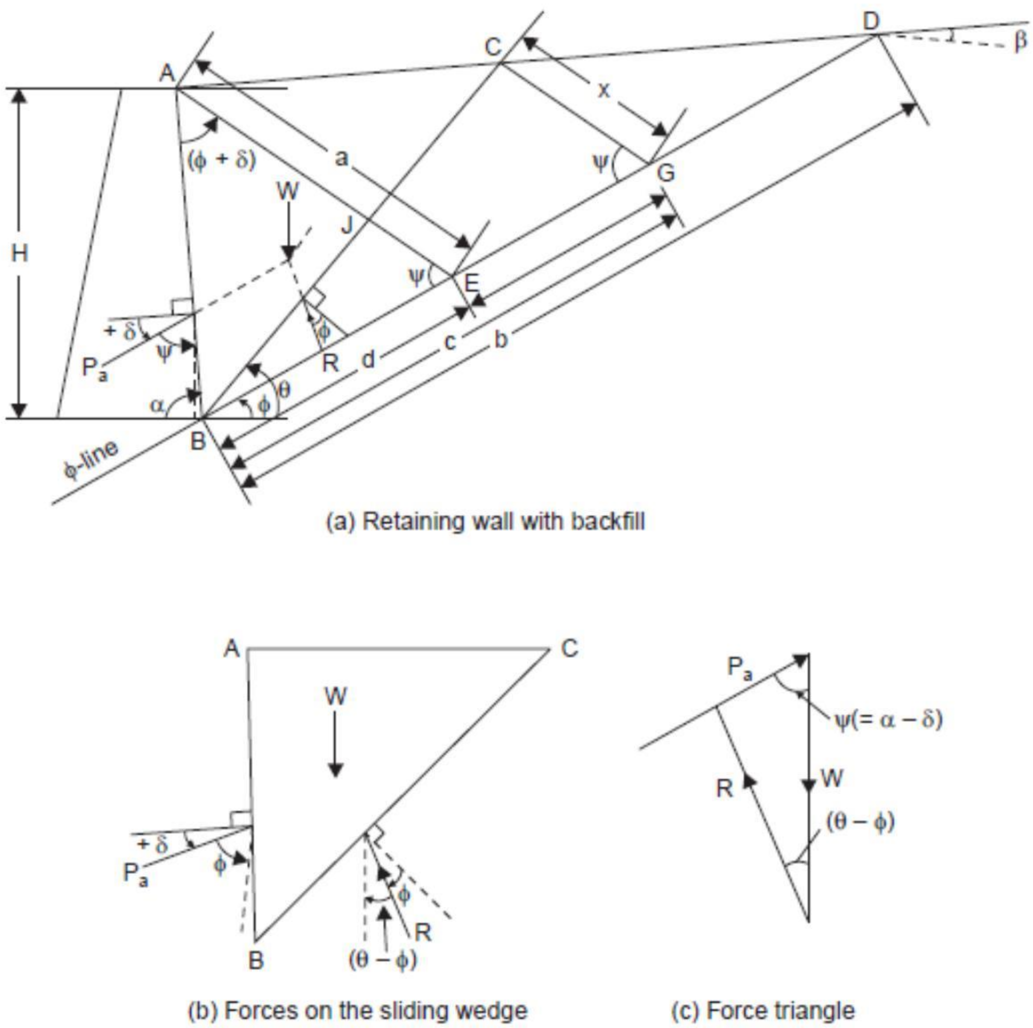


Fig. Rebhann's condition for Coulomb's wedge theory— Location of failure plane for the active case.

Figure (*b*) represents the forces on the sliding wedge and Fig. (*c*) represents the force triangle. Let *BD* be a line inclined at φ to the horizontal through *B*, the heel of the wall, *D* being the intersection of this φ-line with the surface of the backfill. The value of *Pa* depends upon the angle θ relating to the location of the failure plane. *Pa* will be zero when $\theta = \varphi$, and increases with an increase in θ up to a limit, beyond which it decreases and reaches zero again when $\theta = 180° - \alpha$. The situations when *Pa* is zero are both ridiculous, since in the first case, no wall is required to retain a soil mass at an angle φ and in the second, the failure wedge has no mass. Thus, the failure plane will lie between the φ-line and the back of the wall. Let *AE* be drawn at an angle (φ + δ) to the wall face *AB* to meet the φ-line in *E*. Let *CG* be drawn parallel to *AE* to meet the φ-line in *G*. Let the distances be denoted as follows: *AE* = *a* *BG* = *c* *CG* = *x* *BD* = *b* *BE* = *d* It is required to determine the criterion for which *Pa* is the maximum, which is supposed to give the correct location of the failure surface. Weight of the soil in the sliding wedge

W = γ. (*ABC*)

= γ. (Δ*ABD* – *BCD*)

= γ. (*b*/2). (sin ψ) (*a* – *x*)

Value of thrust on the wedge (the same as the thrust on the wall).

$$P_a = \frac{W.x}{c}, \text{ since } \Delta BCG \text{ is similar to the triangle of forces.}$$

$$\therefore \quad P_a = \frac{\gamma bx}{2c}(a - x).\sin\psi \quad \ldots$$

If $\frac{DG}{CG} = k, c = b - kx$

$$\therefore \quad P_a = \frac{\gamma bx}{2(b - kx)}.(a - x)\sin\psi$$

For the value of P_a to be a maximum, $\frac{\partial P_a}{\partial x} = 0$,

since *x* is the only value which varies with the orientation of the failure plane.

$$\therefore \quad \frac{\partial P_a}{\partial x} = (b - kx)(a - 2x) + kx(a - x) = 0$$

$$(a - x)(b - kx + kx) - x(b - kx) = 0$$

$$b(a - x) = cx$$

Multiplying throughout by $\frac{1}{2}\sin\psi$,

$$\frac{1}{2} ba \sin\psi - \frac{1}{2} bx \sin\psi = \frac{1}{2} cx \sin\psi$$

$$\Delta ABD - \Delta BCD = \Delta BCG$$

$$\Delta ABC = \Delta BCG$$

This equation signifies that for *EC* to be the failure plane the requirement is that the area of the failure wedge *ABC* be equal to the area of the triangle *BCG*. This is known as ―Rebhann‘s condition‖, since it was demonstrated first by Rebhann is 1871. The triangles *ABC* and *BCG* which are equal have a common base *BC*; hence their altitudes on to *BC* should be equal; or *AJ*. sin ∠*AJB* = *CG*. sin ∠*BCG* But ∠*AJB* = ∠*BCG* as *CG* is parallel to *AJ*. This leads to *CG* = *AJ* = *x*; and *JE* = *a* – *x* Triangles DAE and DCG are similar.

$$\text{Hence } \frac{(b-d)}{(b-c)}.x = a$$

Also, triangles *BCG* and *BJE* are similar. Consequently,*d/c* .*x* = *a* – *x* Subtracting one from the another,

$$x\left(\frac{b-d}{b-c} - \frac{d}{c}\right) = x$$

Simplifying,

$$c^2 = bd$$

$$c = \sqrt{bd}$$

Thus if *c* is known, the position of *G* and hence that of the most dangerous rupture *p* surface, *BC*, can be determined and the weight of the sliding wedge, *W*, and the active thrust, *Pa*, can be calculated. The relationship expressed by Eq. 13.44 is called the ―Poncelet Rule‖ after Poncelet (1840). It is obvious that Rebhann‘s condition leads one to Poncelet‘s rule and the satisfaction of one of these two implies that of the other automatically.

$$cx = b(a-x)$$

$$x = \frac{ab}{b+c}$$

$$P_a = \frac{1}{2}\gamma x^2.\sin\psi$$

$$\psi = \alpha - \delta$$

$$c = \sqrt{bd}$$

$$x = \frac{ab}{b+c}$$

$$P_a = \frac{1}{2}\gamma x^2.\sin\psi$$

which gives an analytical procedure for the computation of the active thrust by Coulomb‘s wedge theory. However, elegant graphical methods have been devised and are preferred to the analytical approach, in view of their versatility, coupled with simplicity. The graphical method to follow is given by Poncelet and it is also sometimes known as the Rebhann‘s graphical method, since it is based on Reghann‘s condition. The steps involved in the graphical method are as follows, with reference to Fig.

(*i*) Let *AB* represent the backface of the wall and *AD* the backfill surface.

(*ii*) Draw *BD* inclined at φ with the horizontal from the heel *B* of the wall to meet the backfill surface in *D*.

(*iii*) Draw *BK* inclined at $\psi(= \alpha - \delta)$ with *BD,* which is the ψ-line.

(*iv*) Through *A*, draw *AE* parallel to the ψ-line to meet *BD* in *E*. (Alternatively, draw *AE* at $(\varphi + \delta)$ with *AB* to meet *BD* in *E*).

(*v*) Describe a semi-circle on *BD* as diameter.

(*vi*) Erect a perpendicular to *BD* at *E* to meet the semi-circle in *F*.

(*vii*) With *B* as centre and *BF* as radius draw an arc to meet *BD* in *G*.

(*viii*) Through *G*, draw a parallel to the ψ-line to meet *AD* in *C*.

(*ix*) With *G* as centre and *GC* as radius draw an arc to cut *BD* in *L*; join *CL* and also draw a perpendicular *CM* from *C* on to *LG*.

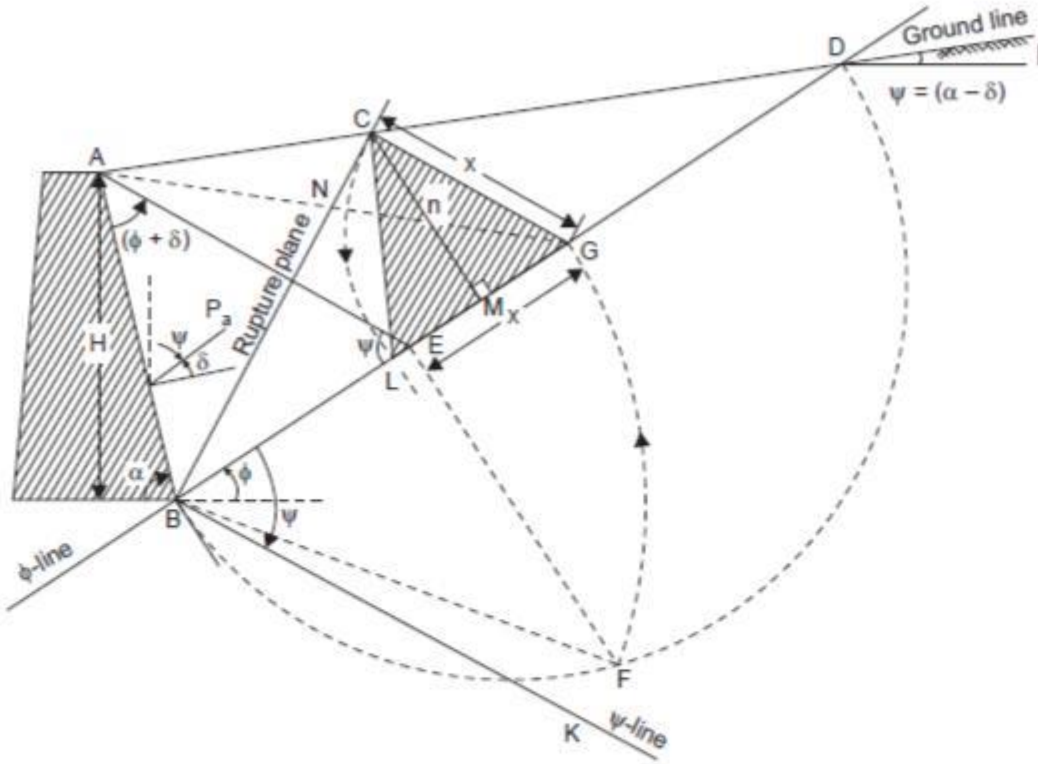


Fig. Poncelet graphical construction for active thrust

BC is the required rupture surface. The criterion may be checked as follows: Since *ABC* = *BCG*, and *BC* is their common base, their altitudes on *BC* must be equal; or *AN* sin ∠*ANB* = *NG* sin ∠*GNC* that is to say *AN* = *NG*, since ∠*ANB* = ∠*GNC*. (*N* is intersection of *AG* and *BC*). Thus, if *AN* and *NG* are measured and found to be equal, the construction is correct. The active thrust

Pa is given by

$$P_a = \frac{1}{2}\gamma x^2 . \sin\psi, \text{ where } CG = LG = x$$

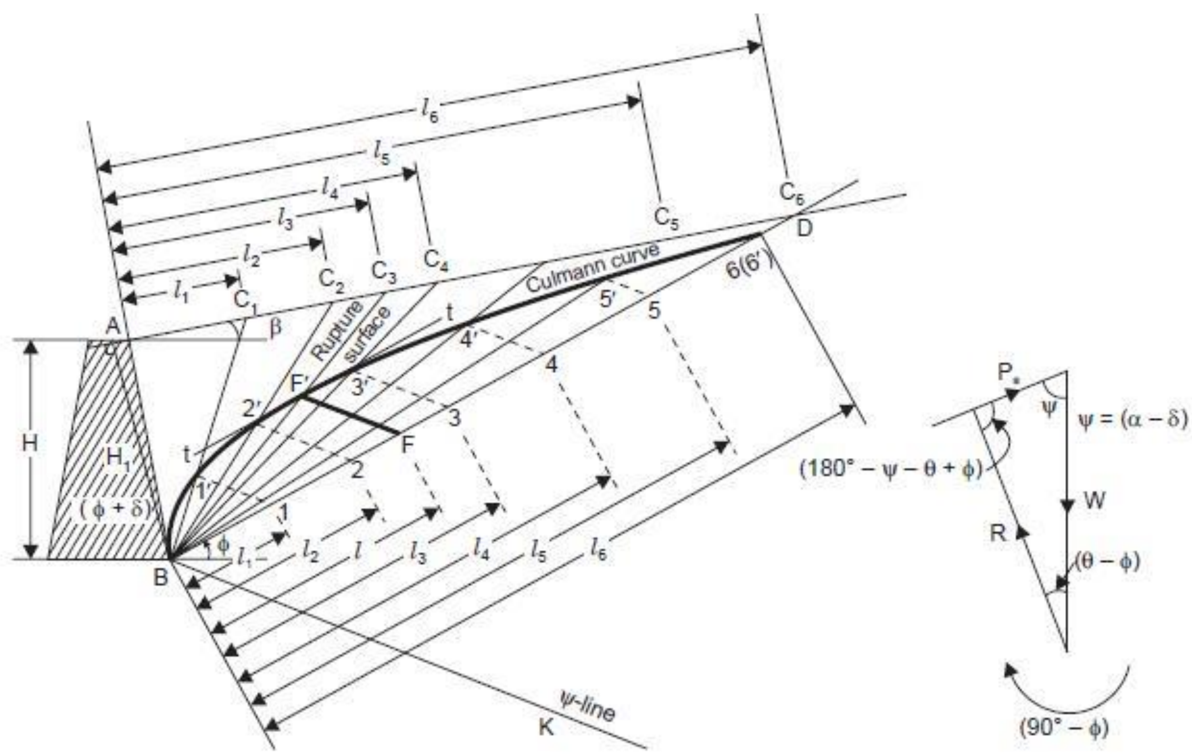
$$= \gamma.(\Delta CGL)$$

$$= \frac{1}{2}\gamma.x.n, \text{ where } n = CM, \text{ the altitude on the } LG.$$

(Incidentally, with the notation of Fig. 13.27 (*a*), it may be easily understood that *W* =1/2 γ.*x*.*n*.

3.8 Culmann's Graphical Method

Karl Culmann (1866) gave his own graphical method to evaluate the earth pressure from Coulomb the earth pressure and to locate the most dangerous rupture surface according to Coulomb's wedge theory. This method has more general application than Poncelet's and is, in fact, a simplified version of the more general trial wedge method. It may be conveniently used for ground surface of any shape, for different types of surcharge loads, and for layered backfill with different unit weights for different layers. With reference to Fig. 13.33 (*b*), the force triangle may be imagined to the rotated clockwise through an angle (90° – φ), so as to bring the vector *W* parallel to the φ-line; in that case, the reaction, *R* will be parallel to the rupture surface, and the active thrust, *Pa*, parallel to the ψ-line.'s theory. Culmann's method permits one to determine graphically the magnitude of



(a) Culmann curve (b) Force triangle

Fig. Culmann's graphical method for active thrust

Hence, if weights of the various sliding wedges arising out of arbitrarily assumed sliding surface are set off to a convenient force scale on the φ-line from the heel of the wall and if lines parallel to the ψ-line are drawn from the ends of these weight vectors to meet the respective assumed rupture lines, the force triangle for each of these sliding wedges will be complete. The end points of the active thrust vectors, when joined in a sequence, form what is known as the ―Culmann-curve‖. The maximum value of the active thrust may be obtained from this curved by drawing a tangent parallel to the φ-line, which represents the desired active thrust, *Pa*. The corresponding rupture surface, which represents the most dangerous rupture surface, may be obtained by the line joining the heel of the wall to the end of the maximum pressure vector.

The steps in the construction may be set out as follows:

(*i*) Draw the ground line, φ-line, and ψ-line, and the wall face *AB*.

(*ii*) Choose an arbitrary failure plane *BC*1. Calculate weight of the wedge *ABC* and plot it as *B*-1 to a convenient scale on the φ-line.

(*iii*) Draw 1 – 1′ parallel to the ψ-line through 1 to meet *BC*1 in 1′. 1′ is a point on the Culmann line.

(*iv*) Similarly, take some more failure planes *BC*2, *BC*3, ..., and repeat the steps (*ii*) and (*iii*) to establish points 2′, 3′, ...

(*v*) Join *B*, 1′, 2′, 3′, etc., smoothly to obtain the Culmann curve.

(*vi*) Draw a tangent *t-t*, to the Culmann line parallel to the φ-line. Let the point of the tangency be *F′*

(*vii*) Draw *F′F* parallel to the ψ-line to meet the φ-line in *F*.

(*viii*) Join *BF′* and produce it to meet the ground line in *C*.

(*ix*) *BF′C* represents the failure surface and *F F ′* represents *Pa* to the same scale as that chosen to represent the weights of wedges. If the upper surface of the backfill is a plane, as shown in Fig., the weights of wedges will be proportional to the distances l_1, l_2 ... (bases), since they have a common-height, *H*1. Thus *B*-1, *B*-2, etc ..., may be made equal or proportional to l_1, l_2, etc. The sector scale may be easily obtained by comparing *BF* with the weight of wedge *ABC*.

Thus $P_a = \frac{\overrightarrow{F'F}}{\overrightarrow{BF}} \frac{1}{2} \gamma(H_1)\, 1. \frac{\overrightarrow{F'F}}{l} = \frac{1}{2}. \gamma H_1(\overrightarrow{EF})$, if the bases themselves are used to represent the weight vector.

Comparison of Coulomb's Theory with Rankine's Theory

The following are the important points of comparison:

(*i*) Coulomb considers a retaining wall and the backfill as a system; he takes into account the friction between the wall and the backfill, while Rankine does not.

(*ii*) The backfill surface may be plane or curved in Coulomb‘s theory, but Rankine‘s allows only for a plane surface.

(*iii*) In Coulomb‘s theory, the total earth thrust is first obtained and its position and direction of the earth pressure are assumed to be known; linear variation of pressure with depth is tacitly assumed and the direction is automatically obtained fromthe concept of wall friction. In Rankine‘s theory, plastic equilibrium inside a semiinfinite soil mass is considered, pressures evaluated, a retaining wall is imagined to be interposed later, and the location and magnitude of the total earth thrust are established mathematically.

(*iv*) Coulomb‘s theory is more versatile than Rankine‘s in that it can take into account any shape of the backfill surface, break in the wall face or in the surface of the fill, effect of stratification of the backfill, effect of various kinds of surcharge on earth pressure, and the effects of cohesion, adhesion and wall friction. it lends itself to elegant graphical solutions and gives more reliable results, especially in the determination of the passive earth resistance; this is inspite of the fact that static equilibrium condition does not appear to be satisfied in the analysis.

(*v*) Rankine‘s theory is relatively simple and hence is more commonly used, while Coulomb‘s

theory is more rational and versatile although cumbersome at times; therefore, the use of the latter is called for in important situations or problems.

Problems

1. A retaining wall, 6 m high, retains dry sand with an angle of friction of 30° and unit weight of 16.2 kN/m^3. Determine the earth pressure at rest. If the water table rises to the top of the wall, determine the increase in the thrust on the wall. Assume the submerged unit weight of sand as 10 kN/m^3.

(*a*) Dry backfill:

$\varphi = 30°$ $H = 6$ m

$K0 = 1 - \sin 30° = 0.5$

(Also $K_0 = 0.5$ for medium dense sand)

$\sigma_0 = K_0 \gamma . H$

$$= \frac{0.5 \times 16.2 \times 600}{1000} \mathrm{N/cm^2}$$
$$= 48.6 \ \mathrm{kN/m^2}$$

Thrust per metre length of the wall = 48.6 ×1/2× 6 = **145.8 kN**

(*b*) Water level at the top of the wall

The total lateral thrust will be the sum of effective and neutral lateral thrusts.

Effective lateral earth thrust, $P_0 = 1/2\ K_0\ \gamma . H^2$

=1/2× 0.5 × 10 × 6 × 6 kN / m.run

= 90 kN/m. run

Neutral lateral pressure $Pw = 1/2 \gamma_w H^2$

≈1/2× 10 × 6 × 6 kN/ m. run

≈ 180 kN/m. run

Total lateral thrust = **270 kN/m. run**

Increase in thrust = **124.2 kN/m. run**

This represents an increase of about **85.2%** over that of dry fill.

2. What are the limiting values of the lateral earth pressure at a depth of 3 metres in a uniform sand fill with a unit weight of 20 kN/m3 and a friction angle of 35°? The ground surface is level. If a retaining wall with a vertical back face is interposed, determine the total active thrust and the total passive resistance which will act on the wall.

Depth, H = 3 m

γ =20 kN/ m^3

$\Phi=35^0$

for sand fill with level surface.

Limiting values of lateral earth pressure:

$$\text{Active pressure} \quad = K_a.\gamma H = \frac{1-\sin 35°}{1+\sin 35°} \times 20 \times 3$$

= 0.271 × 60

= **16.26 kN/m^2**

$$\text{Passive pressure} \quad = K_p.\ \gamma H = \frac{1+\sin 35°}{1-\sin 35°} \times 20 \times 3$$

= 3.690 × 60

= **221.4 kN/m^2**

Total active thrust per metre run of the wall

$$P_a = \frac{1}{2}\gamma H^2 K_a = 16.26 \times \frac{1}{2} \times 3 = \mathbf{24.39\ kN}$$

Total passive resistance per metre run of the wall

$$P_p = \frac{1}{2}\gamma H^2 . K_p = 221.4 \times \frac{1}{2} \times 3 = \mathbf{332.1\ kN}$$

MODULE 4

BEARING CAPACITY OF SHALLOW FOUNDATION:

4.1 TYPES OF FOUNDATIONS,

4.2 DETERMINATION OF BEARING CAPACITY BY TERZAGHI'S AND BIS METHOD (IS: 6403),

4.3 EFFECT OF WATER TABLE AND ECCENTRICITY,

4.4 FIELD METHODS - PLATE LOAD TEST AND SPT

4.5 PROPORTIONING OF SHALLOW FOUNDATIONS- ISOLATED AND COMBINED FOOTINGS (ONLY TWO COLUMNS)

PROPORTIONING SHALLOW FOUNDATIONS

4.11 Introductory concepts on foundations

The ultimate support for any structure is provided by the underlying earth or soil material and, therefore, the stability of the structure depends on it. Since soil is usually much weaker than other common materials of construction, such as steel and concrete, a greater area or volume of soil is necessarily involved in order to satisfactorily carry a given loading. Thus, in order to impart the loads carried by structural members of steel or concrete to soil, a load transfer device is necessary. The structural foundation serves the purpose of such a device. A foundation is supposed to transmit the structural loading to the supporting soil in such a way that the soil is not overstressed and that serious settlements of the structure are not caused. The type of foundation utilised is closely related to the properties of the supporting soil, since the performance of the foundation is based on that of the soil, in addition to its own. Thus, it is important to recognise that it is the soil-foundation system that provides support for the structure; the components of this system should not be viewed separately. The foundation is an element that is built and installed, while the soil is the natural earth material which exists at the site. Since the stability of structure is dependent upon the soil-foundation system, all forces that may act on the structure during its lifetime should be considered. In fact, it is the worst combination of these that must be considered for design. Typically, foundation design always includes the effect of dead loads plus the live loads on the structures. Other miscellaneous forces that may have to be considered result from the action of wind, water, heat ice, frost, earthquake and explosive blasts.

4.11.2 Choice of foundation type and preliminary selection

The type of foundation most appropriate for a given structure depends upon several factors:

(*i*) Function of the structure and the loads it must carry, (*ii*) the subsurface conditions, (*iii*) the cost of the foundation in comparison with the cost of the superstructure. These are the principal factors, although several other considerations may also enter into the picture. They are usually more than one acceptable solution to every foundation problem in view of the interplay of several factors. Judgment also plays an important part. Foundation design is enriched by scientific and engineering developments; however, a strictly scientific procedure may not be possible for practicing the art of foundation design and construction. The following are the essential steps involved in the final choice of the type of foundation:

1. Information regarding the nature of the superstructure and the probable loading is required, at least in a general way.

2. The approximate subsurface conditions or soil profile is to be ascertained.

3. Each of the customary types of foundation is considered briefly to judge whether it is suitable under the existing conditions from the point of view of the criteria for stability—bearing capacity and settlement. The obviously unsuitable types may be eliminated, thus narrowing down the choice.

4. More detailed studies, including tentative designs, of the more promising types are made in the next phase.

5. Final selection of the type of foundation is made based on the cost—the most acceptable compromise between cost and performance. The design engineer may sometimes be guided by

the successful foundations in the neighbourhood. Besides the two well known criteria for stability of foundations—bearing capacity and settlement—the depth at which the foundation is to be placed, is another important aspect. For small loading on good soils, spread footings could be selected. For columns, individual footings are chosen unless they come too close to one another, in which case, combined footings are used. For a series of closely spaced columns or walls, continuous footings are the obvious choice. When the footings for rows of columns come too close to one another, a raft foundation will be the obvious choice. In fact, when the area of all the footings appears to be more than 50 per cent of the area of the structure in plan, a raft should be considered. The total load it can take will be substantially greater than footings for the same permissible differential settlement. In case a shallow foundation does not answer the problem on hand, in spite of choosing a reasonable depth for the foundation, some type of deep foundation may be required. A pier foundation is justified in the case of very heavy loading as in bridges.

Piles, in effect, are slender piers, which are used to bypass weak strata and transmit loading to hard strata below. As an alternative to raft foundation, the economics of bored piles is considered. After the preliminary selection of the type of the foundation is made, the next step is to evaluate the distribution of pressure, settlement, and bearing capacity. Certain guidelines are given in **Table 15.1** with regard to the selection of the type of foundation based on soil conditions at a site. For the design comments it is assumed that a multistorey commercial structure, such as an office building, is to be constructed.

4.11.3 Allowable bearing pressure

Allowable bearing capacity for a given foundation may be (a) to protect the foundation against a bearing capacity failure, or (b) to ensure that the foundation does not undergo undesirable settlement. There are three definitions for the allowable capacity with respect to a bearing capacity failure.

8.4 Factors influencing the selection of depth of footing

The important criteria for deciding upon the depth at which footings have to be installed may be set out as follows:

1. Footings should be taken below the top (organic) soil, miscellaneous fill, debris or muck. If

the thickness of the top soil is large, two alternatives are available: (*a*) Removing the top soil under the footing and replacing it with lean concrete; and (*b*) removing the top soil in an area larger than the footing and replacing it with compacted sand and gravel; the area of this compacted fill should be sufficiently large to distribute the loads from the footing on to a larger area. The choice between these two alternatives, which are shown in Fig. (*a*) and (*b*) will depend upon the time available and relative economy. Footings should be taken below the depth of frost penetration. Interior footings in heated buildings in cold countries will not be affected by frost. The minimum depths of footings from this criterion are usually specified in the load building codes of large cities in countries in which frost is a significant factor in foundation design. The damage due to frost action is caused by the volume change of water in the soil at freezing temperatures. Gravel and coarse sand above water level, containing less than 3% fines, cannot hold water and consequently are not subjected to frost action. Other soils are subjected to frost-heave within the depth of frost penetration. In tropical countries like India, frost is not a problem except in very few areas like the Himalayan region.

3. Footings should be taken below the possible depth of erosion due to natural causes like surface water run off. The minimum depth of footings on this count is usually taken as 30 cm for single and two-storey constructions, while it is taken as 60 cm for heavier construction.

4. Footings on sloping ground be constructed with a sufficient edge distance (minimum 60 cm to 90 cm) for protecting against erosion

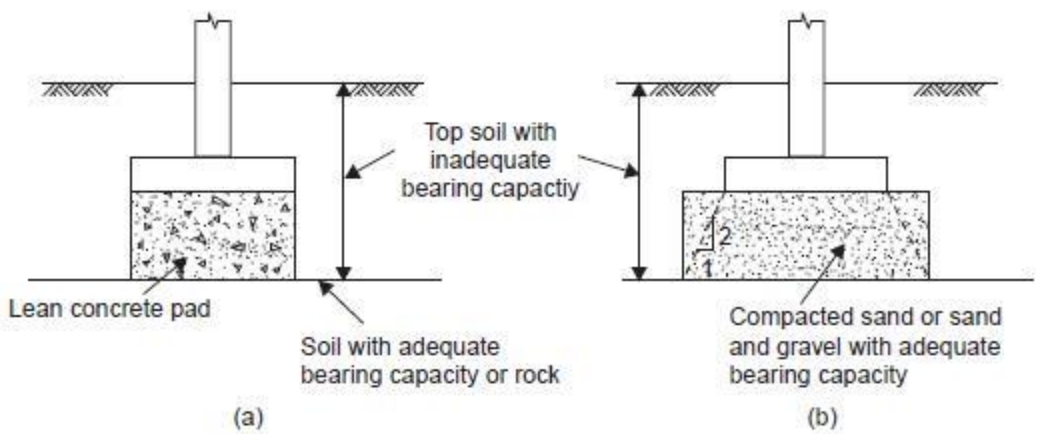


Fig. Alternatives when top soil is of large thickness

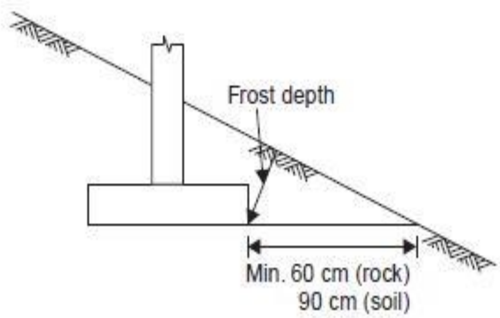


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Fig. Edge distance for floating on sloping ground

5. The difference in elevation between footings should not be so great as to introduce undesirable overlapping of stresses in soil. The guideline used for this is that the maximum difference in elevation should be maintained equal to the clear distance between two footings in the case of rock and equal to half the clear distance between two footings in the case of soil (Fig.). This is also necessary to prevent disturbance of soil under the higher footing due to the excavation for the lower footing.

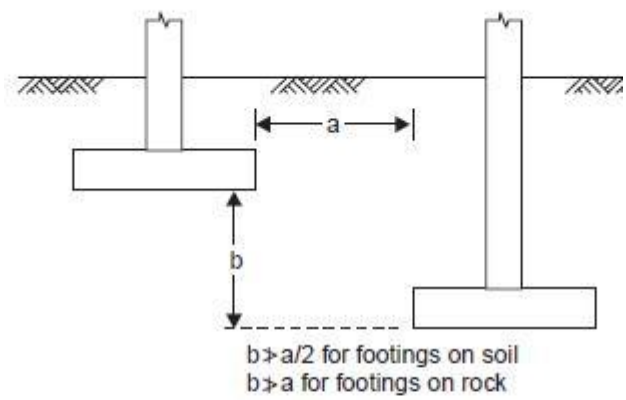

Fig. Footings at different elevations—restrictions

4.11.5 Proportioning of combined footings

The use of combined footings is appropriate either when two columns are spaced so closely that individual footings are not practicable or when a wall column is so close to the property line that it is impossible to center an individual footing under the column. A combined footing is so proportioned that the centroid of the area in contact with the soil lies on the line of action of the resultant of the loads applied to the footing; consequently, the distribution of soil pressure is reasonably uniform. In addition, the dimensions of the footing are chosen such that the allowable soil pressure is not exceeded. When these criteria are satisfied, the footing should neither settle nor rotate excessively. A combined footing may be of rectangular shape or of trapezoidal shape in plan. These are usually constructed using reinforced concrete.

4.11.5.1 Rectangular Combined Footing

A combined footing is usually given a rectangular shape if the rectangle can extend beyond each column the necessary distance to make the centroid of the rectangle coincide with the point at which the resultant of the column loads intersects the base. If the footing is to support an exterior column at the property line where the projection has to be limited, provided the interior column carries the greater load, the length of the combined footing is established by adjusting the projection of the footing beyond the interior column. The width is then obtained by dividing the sum of the vertical loads by the product of the length and the allowable soil pressure. A rectangular combined footing is shown in Fig.

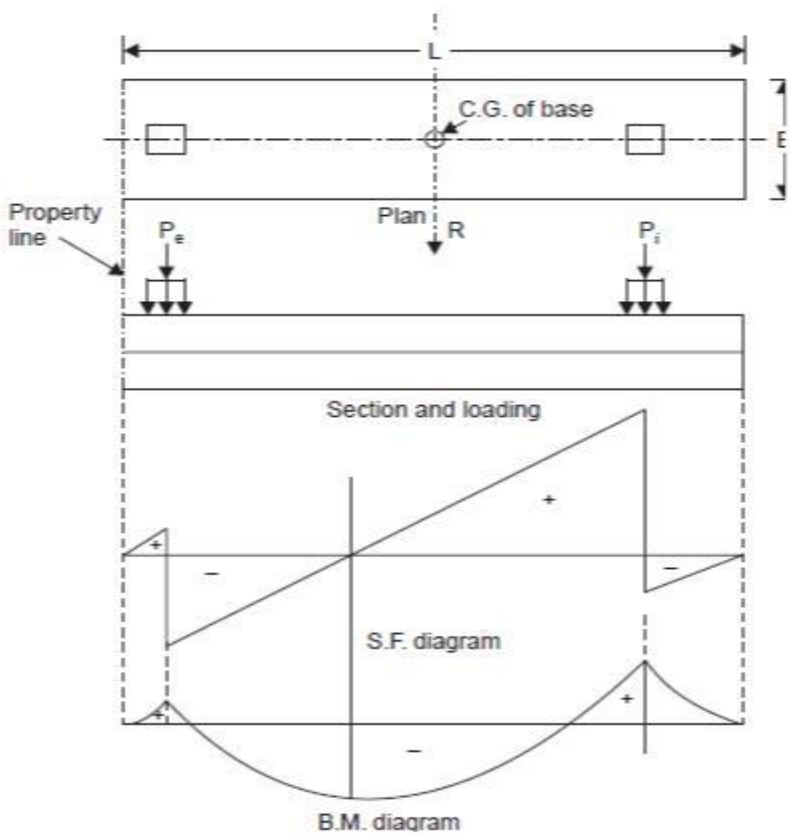

Fig. Rectangular combined footing

4.11.5.2 Trapezoidal Combined Footing

When the two column loads are unequal, the exterior column carrying higher load and when the property line is quite close to the exterior column, a trapezoidal combined footing is used. It may be used even when the interior column carries higher load; but the width of trapezoid will be higher in the inner side. The location of the resultant of the column loads establishes the position of the centroid of the trapezoid. The length is usually limited by the property line at one end and adjacent construction, if any, at the other. The width at either end of the trapezoid can be determined from the solution of two simultaneous equations—one expressing the location of the centroid of the trapezoid and the other equating the sum of the column loads to the product of the allowable soil pressure and the area of the footing. The resulting pressure distribution is linear or uniformly varying (and not uniform) as shown in Fig.

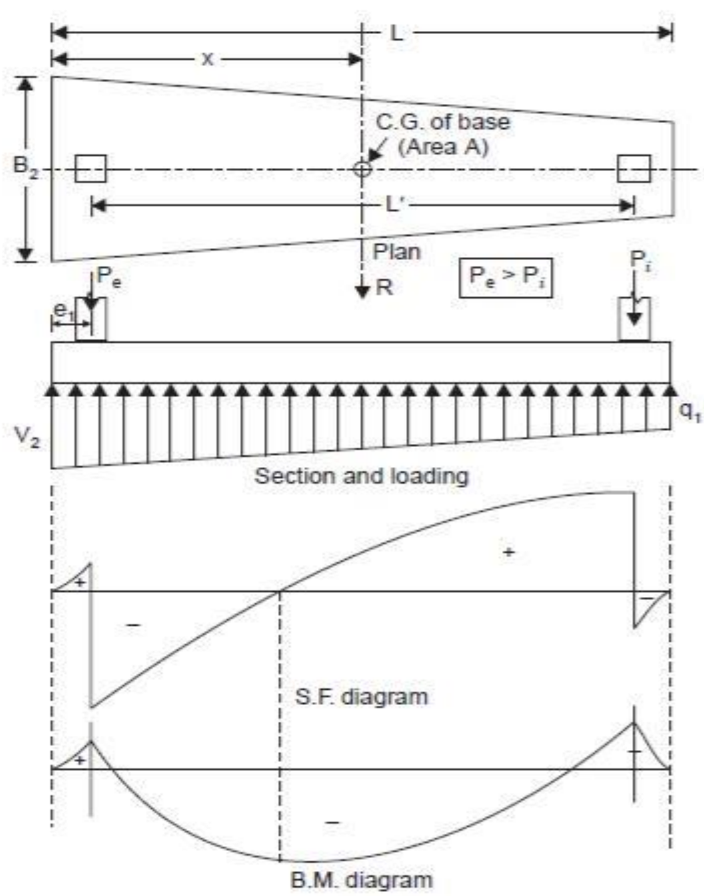

Fig. Trapezoidal combined footing

Problems

1. Compute the ultimate laod that an eccentrically loaded square footing of width 2.1 m with an eccentricity of 0.35 m can take at a depth of 0.5 m in a soil with γ = 18 kN/m^3, *c* = 9 kN/m^2 and φ = 36°, N_c = 52; N_q = 35; and N_γ = 42.

Conventional approach (Peck, Hanson and Thornburn, 1974): For φ = 36°, *Nc*= 52 *Nq* = 35 *N*γ = 42 q_{ult} for axial loading = $1.3cN_c + \gamma D_f N_q + 0.4\gamma_b N_\gamma$ = 1.3 × 9 × 52 + 18 × 0.5 × 35 + 0.4 × 18 × 2.1 × 42 = 608.4 + 315 + 635.03 ≈ 1558 kN/m^2 Eccentricity ratio, *e/b* = 0.35/2.10 = 1/6. If the ultimate load is Q_{tul},

$$\text{maximum soil pressure} = 2 \,.\, q_{av} = \frac{2 \times Q_{ult}}{\text{Area}} = \frac{2 \times Q_{ult}}{2.1 \times 2.1}$$

$$\text{Equating } q_{ult} \text{ to this value, } 1558 = \frac{2Q_{ult}}{4.41}$$

$$\therefore \quad Q_{ult} = 1558 \times \frac{4.41}{2} \approx 3435 \text{ kN}$$

Useful width concept: *b*′ = *b* – 2*e* = 2.10 – 2 × 0.35 = 1.40 m Since the eccentricity is about only one axis, effective area = 1.40 × 2.10 = 2.94 m2

∴ *q*ult = 1.3 × 9 × 52 + 18 × 0.5 × 35 + 0.4 × 18 × 1.4 × 42 = 608.4 + 315 + 423.36 ≈ 1347 kN/m^2

∴ *Q*ult = *q*ult × effective area = 1347 × 2.94 ≈ **3960 kN.**

There appears to be significant difference between the result obtained by the two methods. The conventional approach is more conservative.

2.Proportion a strap footing for the following data:

Allowable pressures:

150 kN/m2 for *DL* + reduced *LL*

225 kN/m2 for *DL* + *LL*

Column loads

	Column A	Column B
DL	540 kN	690 kN
LL	400 kN	810 kN

Proportion the footing for uniform pressure under DL + reduced LL. Distance c/c of columns = 5.4 m Projection beyond column A not to exceed 0.5 m.

DL + reduced *LL*:

for column A ... 740 kN

for column B ... 1095 kN

Footing A

Assume a width of 2.4 m. Eccentricity of column load with respect to the footing = (1.2 – 0.5) = 0.7 mc/c of footings (assuming footing B to be centrally placed with respect to column B) = 5.4 – 0.7 = 4.7 m

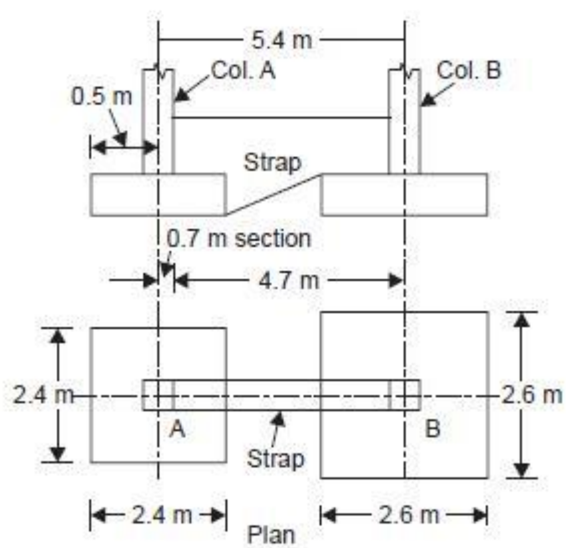

Fig. Strap footing

Enhanced load = 740 ×5 4/47kN = 850 kN

Area required = 850/150 = 5.67 m^2

Width: 5.67/2.40 = 2.36 m.

Use **2.4 m** × **2.4 m** footing (actual area 5.76 m^2).

Footing B

Load on Column B = 1095 kN

Net load = 1095 – 740 ×0 7/4 7 = 985 kN.

Area required = 950/150 = 6.57 m^2

Use **2.6 m** × **2.6 m** footing (actual area 6.76 m^2)

Soil pressure under *DL* + *LL*:

Footing A: Load = 940 ×54/ 4 7 = 1080 kN Pressure =1080/ 5.76 = 187.5 kN/m^2

Footing B: Load = 1500 kN – 940 ×0 7/ 4 7 = 1360 kN Pressure = 1360/ 6.76 ≈ 201 kN/m^2 These are less than 225 kN/m^2.

Hence O.K.

4.11 Assignment Questions

- What is bearing capacity of soil and define various types of bearing capacity
- Explain Terzaghis method of determination of bearing capacity of soils
- Explain proportioning of isolated footing and combined footing

4.12 Outcomes

Students should be able to

- Estimate bearing capacity of soils
- Understand types of foundations
- Design of Shallow foundations

4.13 Further reading

- https://theconstructor.org › Geotechnical Engineering

MODULE-5

PROPORTIONING PILE FOUNDATIONS

Structure

5.0 Objectives

- To understand piles
- To design single pile
- To design group of piles

5.1 Pile Foundations

Deep foundations are employed when the soil strata immediately beneath the structure are not capable of supporting the load with tolerable settlement or adequate safety against shear failure. Merely extending the level of support to the first hard stratum is not sufficient, although this is a common decision that is reached. Instead, the deep foundation must be engineered in the same way as the shallow foundation so that the soil strata below remain safe and free of deleterious settlement. Two general forms of deep foundation are recognized.

1. Pile foundation

2. Pier, caisson or well foundation.

Piles are relatively long, slender members that are driven into the ground or cast-insitu. Piers, caissons or wells are larger, constructed by excavation and are sunk to the required depth; these usually permit visual examination of the soil or rock on which they rest. In effect they are deep spread footings or mats. They are normally used to carry very heavy loads such as those from bridge piers or multi-storeyed buildings. A sharp distinction between piles and piers is impossible because some foundations combine features of both. Piles have been used since prehistoric times. The Neolithic inhabitants of Switzerland, 12,000 years ago, drove wooden poles in the soft bottoms of shallow lakes and on them erected their homes, high above marauding animals and warring neighbours. Pile foundations were used by Romans; Vitruvius (59 A.D.) records the use of such foundations. Today, pile foundations are much more common than any other type of deep foundation, where the soil conditions are unfavourable.

5.2 Classification of piles

Piles may be classified in a number of ways based on different criteria:

(*a*) Function or action

(*b*) Composition and material

(*c*) Installation

5.2.1 Classification Based on Function or Action

Piles may be classified as follows based on the function or action:

End-bearing piles

Used to transfer load through the pile tip to a suitable bearing stratum, passing soft soil or water.

Friction piles

Used to transfer loads to a depth in a frictional material by means of skin friction along the surface area of the pile.

Tension or uplift piles

Used to anchor structures subjected to uplift due to hydrostatic pressure or to overturning moment due to horizontal forces.

Compaction piles

Used to compact loose granular soils in order to increase the bearing capacity. Since they are not required to carry any load, the material may not be required to be strong; in fact, sand may be used to form the pile. The pile tube, driven to compact the soil, is gradually taken out and sand is filled in its place thus forming a ‘sand pile’.

Anchor piles

Used to provide anchorage against horizontal pull from sheetpiling or water.

Fender piles

Used to protect water-front structures against impact from ships or other floating objects.

Sheet piles

Commonly used as bulkheads, or cut-offs to reduce seepage and uplift in hydraulic structures.

Batter piles

Used to resist horizontal and inclined forces, especially in water front structures.

Laterally-loaded piles

Used to support retaining walls, bridges, dams, and wharves and as fenders for harbor construction.

5.2.2 Classification Based on Material and Composition

Piles may be classified as follows based on material and composition:

Timber piles

These are made of timber of sound quality. Length may be up to about 5 m; splicing is adopted for greater lengths. Diameter may be from 30 to 40 cm. Timber piles perform well either in fully dry condition or submerged condition. Alternate wet and dry conditions reduce the life of a timber pile; to overcome this, creosoting is adopted. Maximum design load is about 250 kN.

Steel piles

These are usually H-piles (rolled H-shape), pipe piles, or sheet piles (rolled sections of regular shapes). They may carry loads up to 1000 kN or more.

Concrete piles

These may be ‘precast’ or ‘cast-in-situ’. Precast piles are reinforced to withstand handling stresses. They require space for casting and storage, more time to cure and heavy equipment for handling and driving.

Cast-in-situ piles are installed by pre-excavation, thus eliminating vibration due to driving and handling. The common types are Raymond pile, Mac Arthur pile and Franki pile.

Composite piles

These may be made of either concrete and timber or concrete and steel. These are considered suitable when the upper part of the pile is to project above the water table. Lower portion may be of untreated timber and the upper portion of concrete. Otherwise, the lower portion may be of steel and the upper one of concrete.

5.2.3 Classification Based on Method of Installation

Piles may also be classified as follows based on the method of installation:

Driven piles

Timber, steel, or precast concrete piles may be driven into position either vertically or at an inclination. If inclined they are termed ‘batter’ or ‘raking’ piles. Pile hammers and pile-driving equipment are used for driving piles.

Cast-in-situ piles

Only concrete piles can be cast-in-situ. Holes are drilled and these are filled with concrete. These may be straight-bored piles or may be ‘under-reamed’ with one or more bulbs at intervals. Reinforcements may be used according to the requirements.

Driven and cast-in-situ piles

This is a combination of both types. Casing or shell may be used. The Franki pile falls in this category.

5.3 Use of piles

The important ways in which piles are used are as follows:

(*i*) To carry vertical compressive loads,

(*ii*) To resist uplift or tensile forces, and

(*iii*) To resist horizontal or inclined loads.

(iv) To resist the applied moments in quays

Bearing piles are used to support vertical loads from the foundations of buildings and bridges. The load is carried either by transferring to the incompressible soil or rock below through soft strata, or by spreading the load through soft strata that are incapable of supporting concentrated loads from shallow footings. The former type is called point or end-bearing piles, while the latter are known as friction-piles. Tension piles are used to resist upward forces in structures subjected to uplift, such as buildings with basements below the ground water level, aprons of dams or buried tanks. They are also used to resist overturning of walls and dams and for anchors of towers, guy wires and bulkheads. Laterally loaded piles support horizontal or inclined forces such as the foundations of retaining walls, bridges, dams, and wharves and as fenders in harbour construction. In case the lateral loads are of large magnitude they may be more effectively resisted by batter piles, driven at an inclination. Closely spaced piles or thin sheet piles are used as cofferdams, seepage cut-offs and retaining walls. Piles may be used to compact loose granular soils and also to safeguard foundations against scouring. These are illustrated in Fig.

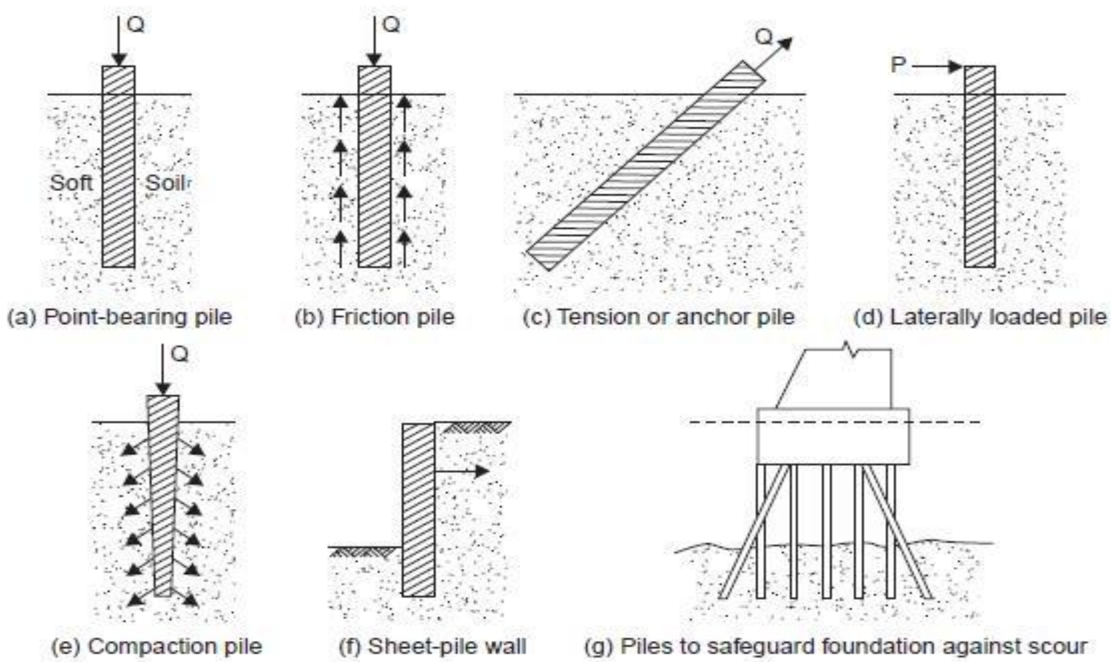

Fig. Uses of piles

5.4 Pile load capacity

The ultimate bearing capacity of a pile is the maximum load which it can carry without failure or excessive settlement of the ground. The allowable load on a pile is the load which can be imposed upon it with an adequate margin of safety; it may be the ultimate load divided by a suitable factor of safety, or the load at which the settlement reaches the allowable value. The bearing capacity of a pile depends primarily on the type of soil through which and/ or on which it rests, and on the method of installation. It also depends upon the cross-section and length of the pile. The pile shaft is a structural column that is fixed at the point and usually restrained at the top. The elastic stability of piles, or their resistance against buckling, has been investigate both theoretically and by load tests (Bjerrum, 1957). Both theory and experience demonstrate that buckling rarely occurs because of the effective lateral support of the soil; it may occur only in extremely slender piles in very soft clays or in piles that extend through open air or water. Therefore, the ordinary pile in sand or clay may be designed as though it were a short column. The pile transfers the load into the soil in two ways. Firstly, through the tip-in compression, termed 'end-bearing' or 'point-bearing'; and, secondly, by shear along the surface, termed 'skin friction'. If the strata through which the pile is driven are weak, the tip resting on a hard stratum transfers most part of the load by end-bearing; the pile is then said to be an end bearing pile. Piles in homogeneous soils transfer the greater part of their load by skin friction, and are then called friction piles; however, nearly all piles develop both end-bearing and skin friction.

The following is the classification of the methods of determining pile capacity:

(*i*) Static analysis (*ii*) Dynamic analysis

(*iii*) Load tests on pile (*iv*) Penetration tests

Problems

1. A timber pile was driven by a drop hammer weighing 30 kN with a free fall of 1.2 m. The average penetration of the last few blows was 5 mm. What is the capacity of the pile according to Engineering News Formula?

Allowable load on the pile

$$Q_{ap} = \frac{500\, W_h \,.\, H}{3(s + 25)} \text{ for drop hammer,}$$

H being in metres and s being in mm. W_h = 30 kN H = 1.2 m s = 3 mm

$$Q_{ap} = \frac{500 \times 3 \times 1.2}{3 \times (5 + 25)}\ t = \mathbf{200\ kN.}$$

2. A pile is driven with a single acting steam hammer of weight 15 kN with a free fall of 900 mm. The final set, the average of the last three blows, is 27.5 mm. Find the safe load using the Engineering News Formula.

Allowable load on the pile

$$Q_{ap} = \frac{500\ W_h\ .\ H}{3(s + 2.5)} \text{ for steam hammer,}$$

H being in metres and s in mm.

Wh = 15 kN H = 900 mm = 0.9 m

$$Q_{ap} = \frac{500 \times 15 \times 0.9}{3(27.5 + 2.5)} \text{ kN} = \mathbf{75\ kN.}$$

3. A reinforced cement concrete pile weighing 30 kN (including helmet and dolly) is driven by a drop hammer weighing 30 kN with an effective fall of 0.9 m. The average penetration per blow is 15 mm. The total temporary elastic compression of the pile, pile cap and soil may be taken as 15 mm. Coefficient of restitution 0.36. What is the allowable load on the pile with a factor of safety of 2? Use Hiley's formula.

$$W_p = 30 \text{ kN} \qquad W_h = 30 \text{ kN} \quad R = \frac{W_p}{W_h} = 1$$

Effective fall = ηH = 0.9 m = 900 mm

s = 15 mm C =1/2 (total elastic compression of pile, pile cap and soil)

C_r = 0.36 =1/2 × 15 = 9 mm

$$Q_{ap} = \frac{Q_{up}}{2} = \frac{1}{2}\left[\frac{W_h \,.\, H \,.\, \eta}{(s+C)}\left(\frac{1+C_r{}^2 \,.\, R}{1+R}\right)\right]$$

$$= \frac{1}{2}\left[\frac{30\times 900\times 1}{(15+9)}\times\frac{1+(0.36)^2\times 1}{(1+1)}\right]$$

$$= \frac{1}{2}\,.\,\frac{30}{24}\,.\,900\times\frac{1.1296}{2}$$

$$= 317.7 \text{ kN}$$

Approximate safe load may be taken as
315 kN.

5.5 Assignment Questions

- What is a pile? Explain pile group capacity
- Derive expression for capacity of a single pile
- Derive expression for capacity of a pile group

5.6 Outcomes

Students should be able to

- Decide upon soil exploration techniques to be adopted for different site condition
- Conduct soil exploration and to do the report of the same
- Collect soil sample by using proper sampling technique base on requirement
- Understand dewatering techniques and efficiency of lowering of water table

5.7 Further reading

- https://theconstructor.org/geotechnical/foundations/pile/